ENGG. PHYSICS
LECTURE NOTES
BY

SATYA NARAYAN TRIPATHY,
Lect. In PHYSICS, GP Koraput

Introduction To Physics

Physics → Physics is the branch of Science which deals w the study of nature and laws.

There are numerous number of events taking place in the nature such as falling of fruits from trees, rotation of pla around the sun, blue Colour of sky, ~~lighting~~ lightening a thunder in rainy season and so on.

All these events are taking place according to some basic laws of nature. Revealing these laws of nature from observed events is also called as physics.

Physical Quantities

There are large number of physical quantities that ~~are~~ u in physics to explain the events taking place in the nature

These physical quantities are categorised into two groups

i) Fundamental Physical quantities / Base quantities

ii) Derived Physical quantities.

Fundamental Physical quantities

The physical quantities are said to be fundamental if an only if they satisfy following points.

a) The fundamental physical quantities are independent of each other

b) All other quantities may be expressed in terms ~~and~~ of fundamental physical quantities.

There are seven fundamental physical quantities in physics, which are listed below.

Sl No.	Name of the fundamental Physical quantity	Symbol
1.	Mass	M
2.	Length	L
3.	Time	T
4.	Electric Current	I
5.	Temperature	K
6.	Amount of Substance	Mole / Mol
7.	Luminous Intensity / Luminosity	~~cd~~

Derived Physical Quantities
Derive physical quantities are the physical quantities which can be obtained from the fundamental physical quantities by multiplication and/or division

Examples :- Area , Volume , Velocity, acceleration, force, pressure etc.

Unit
In ~~odor~~ order to measure physical quantity, We need some standard units of these quantities.
Thus, a physical quantity needs a unit for its quantitively expression.

System of Units
There are several system of units as follow :-
i) MKS System
In this system of unit, the unit of length is meter, the unit of mass is kilogram, and the unit of time is second.

ii) CGS system
In this system of units, the unit of length is Centimete the unit of Mass is gram and the unit of time is second.

iii) International System of units / S.I units.
In order to include the units of all the physical quantity used in the physics, a new system of unit introduced and is called SI units are International System of units.

Fundamental unit
The SI units defined for the fundamental physical quantiti are called fundamental units.

SL No.	Name of the fundamental Physical quantity	SI unit	Symbol
1	Mass	Kilogram	kg
2	Length	Metere	M
3	Time	Second	S
4	Temperature	Kelvin	K
5	Electric Current	Ampere	A
6	Amount of Substance	Mole	Mol
7.	Luminosity	Candela	Cd

Unit Conversion
Ex - $50\,cm = ?\,m$

$1\,m = 100\,cm$

$1\,cm = \frac{1}{100}\,m$

Ans $50\,cm = 50 \times \frac{1}{100}\,m$

$= 5\cancel{0} \times \frac{1}{10\cancel{0}} = \frac{5}{10} = 0.5\,m.$

② $20\,m = \_\_\_\_\_\_\,cm$

$1\,m = 100\,cm$

$20\,m = 20 \times 100$

$= 2000\,cm$

⑤ $1\,min = 60\,s$

$1\,hr = 3600\,s$

$1\,s = \frac{1}{60}\,min$

$1\,s = \frac{1}{3600}\,h$

3 $5\,kg = \_\_\_\_\_\_\,g$

$1\,kg = 1000\,g$

$5\,kg = 5 \times 1000$

$= 5000\,g$

4 $3\,gm = \_\_\_\_\_\_\,kg$

$1\,gm = 1/1000\,kg$ or $0.001\,kg$

$3\,gm = 3/1000\,kg$ or $0.003\,kg$

Problem - 1

A body is moving with a velocity 36 km/hr. What is the value of velocity in m/s.

$36\,km/hr = \frac{3\cancel{6} \times 100\cancel{0}}{\cancel{360}\cancel{0}} = 10\,m/s.$

Problem-2
Convert a velocity of 20 m/s in km/hr.

$$20 \times \frac{\frac{1}{1000}}{\frac{\cancel{36}\ 1}{3600}}$$

$$20 \times \frac{1}{10\cancel{00}} \times 36\cancel{00}$$

72 km/h

CHAPTER - II

Dimensions ~~and Vectors~~

Dimension

When a physical quantity is expressed in terms of the fundamental physical quantity, it is written as the product of different power of the fundamental quantities. These powers are called dimensions of the physical quantity.

Defination of dimensions

The dimension of a physical quantity may be defined as the powers to which the fundamental physical quantities must be raised to represent the physical quantity.

Explanation

Consider a physical quantity named density

We have,

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}}$$

$$\text{Density} = \frac{\text{Mass}}{(\text{length})^3}$$

$$\rho = \frac{M}{L^3}$$

$$\text{or, } \rho = M^1 L^{-3} \quad \text{——} (1)$$

Dimensions of ρ are 1 in M and -3 in L.

Note

1) In dimensional analysis, the physical quantity whose dimensions to be obtained is written in Square bracket.

$\therefore$ Eq^n -(1) is written as

$$[\rho] = M^1 L^{-3} \quad -(2)$$

2) The expression of a physical quantity in terms of fundamental physical quantity is called dimensional formula of that quantity.

$\therefore$ The dimensional formula of ρ is $M^1 L^{-3}$.

Problem - 1

Find the dimensions and dimensional formula of the following physical quantity.

i) Area
ii) Volume
iii) Velocity
iv) Acceleration
v) Force
vi) Momentum

Sol^n

Area = length X length

$$[\text{Area}] = L^2$$

Dimension of Area is 2 in L and DF is L^2.

ii) Volume = length × length × length

$[Volume] = L^3$

Dimension of volume is 3 in L and DF is L^3.

iii) $Velocity = \frac{Distance}{Time}$ [∴ Distance = Length]

$\Rightarrow \frac{Length}{Time}$

$[Velocity] = \frac{L}{T}$

$[velocity] = L^1 T^{-1}$

Dimension of velocity is 1 in L and -1 in T and DF is L^1T^{-1}

iii) $Acceleration = \frac{Velocity}{Time}$

We know $[Velocity] = L^1T^{-1}$

So, the eqn can be written as; $[acceleration] = \frac{L^1T^{-1}}{T}$

$\Rightarrow [Acceleration] = LT^{-2}$

Dimension of acceleration is 1 in L and -2 in T and DF is LT^{-2}

iv) Force = Mass × Acceleration

We know $[Acceleration] = LT^{-2}$

$[Force] = M \times LT^{-2}$

$[Force] = MLT^{-2}$

Dimension of force is 1 in M, 1 in L and −2 in T and DF is MLT^{-2}

vi) Momentum = Mass X Velocity

We know [Velocity] = $L^{1}T^{-1}$

$[\text{Momentum}] = M \times LT^{-1}$
$[\text{Momentum}] = MLT^{-1}$

Dimension of Momentum is 1 in M, 1 in L and −1 in T and DF is MLT^{-1}

Problem - 2
Find the dimensional formulae of the following physi quantity.

i) Pressure
ii) Work
iii) Kinetic energy
iv) Potential energy
v) Impulse

Sol^n
i) Pressure = $\frac{\text{Force}}{\text{Area}}$

We know [Force] = MLT^{-2}
[Area] = L^2

$\therefore$ [Pressure] = $\frac{MLT^{-2}}{L^2}$

$[Pressure] = ML^{-1}T^{-2}$

Dimension of pressure is 1 in M, -1 in L and -2 in T and DF is $ML^{-1}T^{-2}$.

ii) Work = Force X Distance

We know, $[Force] = MLT^{-2}$
$[Distance] = L$

$[Work] = MLT^{-2} \times L$
$[Work] = ML^2T^{-2}$

Dimension of work is 1 in M, 2 in L and -2 in T and DF is ML^2T^{-2}.

iii) Kinetic Energy (KE) = $\frac{1}{2}$ Mass X $(Velocity)^2$

In this formulae a digit is present $\frac{1}{2}$ so this digit is consider as dimensional less ~~digits~~.

So, (KE) = Mass x $(Velocity)^2$

We know $[Mass] = M$
$[Velocity] = LT^{-1}$
$[Velocity]^2 = L^2T^{-2}$

~~Velo~~ $[Kinetic\ Energy] = M \times L^2T^{-2}$
$[Kinetic\ Energy] = ML^2T^{-2}$

Dimension of ~~work is~~ Kinetic Energy is 1 in M, 2 in L and -2 in T and DF is ML^2T^{-2}.

iv) Potential Energy = Mgh $\left[\begin{array}{l} \because g = \text{acceleration due to gravi} \\ h = \text{height} \Rightarrow \text{Length} \end{array}\right.$

[Potential Energy] = Mass X Acceleration X Length
[Potential Energy] = $M \times LT^{-2} \times L$
[Potential Energy] = $\cancel{M \times L^2}$ ML^2T^{-2}

Dimension of potential energy is 1 in M, 2 in L and -2 in T and DF is ML^2T^{-2}.

v) Impulse = Force X Time

We know [Force] = MLT^{-2}

[Impulse] = $MLT^{-2} \times T$
[Impulse] = MLT^{-1}

Dimension of Impulse is 1 in M, 1 in L and -1 in T and DF is MLT^{-1}.

Problem -3

Find the dimensional formulae of the following physical quantit

i) Frequency
ii) Stress
iii) Surface tension
iv) Charge
v) Electric potential
vi) Resistance
vii) Capacitance

Solⁿ

i) Frequency $(f) = \frac{1}{T}$

$[f] = T^{-1}$

Dimension of frequency is -1 in T and dimension formulae is T^{-1}.

ii) Stress $= \frac{\text{Force}}{\text{Time}}$

We know $[\text{Force}] = MLT^{-2}$

$[\text{Time}] = T$

So, $[\text{Stress}] = \frac{MLT^{-2}}{T}$

$[\text{Stress}] = MLT^{-3}$

Dimension of Stress is 1 in M, 1 in L and -3 in T and dimension formulae is MLT^{-3}.

iii) Surface tension $= \frac{\text{Force}}{\text{Length}}$

We know $[\text{Force}] = M^1LT^{-2}$

$[\text{Length}] = L$

So, $[\text{Surface tension}] = \frac{M^1LT^{-2}}{L}$

~~So~~, [Surface tension] = $M^1 L^0 T^{-2}$

Dimension of surface tension is 1 in M, 0 in L and -2 in T and dimensional formulae is $M^1 L^0 T^{-2}$.

iv) Charge = Electric Current X Time

[Charge] = I X T
[Charge] = IT

Dimension of ~~surface~~ charge is 1 in I and 1 in T and DF is IT.

v) Electric potential = $\frac{\text{Work}}{\text{Charge}}$

We know [Work] = $M^1 L^2 T^{-2}$
& [Charge] = IT

So, [Electric potential] = $\frac{M^1 L^2 T^{-2}}{IT}$

[Electric potential] = $M^1 L^2 T^{-3} I^{-1}$

Dimension of electric potential is 1 in M, 2 in L, -3 in T and -1 in I and DF is $M^1 L^2 T^{-3} I^{-1}$.

vi) Resistance = $\frac{\text{electric potential}}{\text{electric current}}$

We know [electric potential] = $M^1 L^2 T^{-3} I^{-1}$

So, ~~electric po~~ $[\text{Resistance}] = \dfrac{M^1L^2T^{-3}I^{-1}}{I}$

$[\text{Resistance}] = M^1L^2T^{-3}I^{-2}$

Dimension of resistance is 1 in M, 2 in L, -3 in T and -2 in I and DF is $M^1L^2T^{-3}I^{-2}$.

vii) Capacitance $= \dfrac{\text{Charge}}{\text{electric potential}}$

We know $[\text{Charge}] = IT$

$[\text{electric potential}] = M^1L^2T^{-3}I^{-1}$

$[\text{Capacitance}] = \dfrac{IT}{M^1L^2T^{-3}I^{-1}}$

$[\text{Capacitance}] = M^{-1}L^{-2}T^4I^2$

Dimension of capacitance is -1 in M, -2 in L, 4 in T and 2 in I and DF is $M^{-1}L^{-2}T^4I^2$.

Principle of homogeniety

Statement :- According to this principle, a physical equation is dimensionally correct if and only if the dimensions of each and every term on both sides of equation must be the same.

Explanation :- Consider a physical equation, $a+b=c$

According to this principle, the above equation will be dimensionally correct if $[A]=[B]=[C]$.

Applications of the dimensional analysis.

There are following applications of dimensional analysis:-

i) To check the correctness of a given physical relation.

ii) To convert the value of a physical quantity from one system of unit to other.

iii) To derive relation between various physical quantities.

To check the correctness of a given physical relation.

Using dimensional analysis and applying principle of homoge the correctness of a physical relation can be checked.

For e.g.,

So, consider a relation $S = ut + \frac{1}{2}at^2$

u = Initial velocity (speed)

S = Displacement (distance)

t = time

a = acceleration

$$[S] = [L]$$

$$[ut] = [LT^{-1} \times T] = [LT^{-1+1}] = [LT^0] = [L]$$

$$\left[\frac{1}{2}at^2\right] = [at^2] = [a] \times [t^2] = [LT^{-2}] \times [T^2] = [LT^{-2+2}] = [LT^0] = [L]$$

Since, dimension of $[S] = [ut] = \left[\frac{1}{2}at^2\right]$, the given relation is correct.

Problem - 4

Checking the correctness of the following relations.

i) $v^2 - u^2 = 2as$ [v = final ~~velo~~ velocity]

ii) $V = \cancel{ut}\ u + at$

iii) $T = 2\pi\sqrt{\dfrac{g}{l}}$

iv) $F = 2\pi\sqrt{\dfrac{g}{l}}$

v) $V = \cancel{f}\ f\lambda$

Solⁿ

i) $v^2 - u^2 = 2as$

$[v^2] = [L^2 T^{-2}]$

$[u^2] = [L^2 T^{-2}]$

$$
\begin{aligned}
[2as] &= [as] \\
&= [a] \times [s] \\
&= [LT^{-2}] \times [L] \\
&= [L^2 T^{-2}]
\end{aligned}
$$

Since, dimension of $[v^2] = [u^2] = [2as]$, the given relation is correct.

ii) $V = u + at$

$[V] = [LT^{-1}]$

$[u] = [LT^{-1}]$

$$[at] = [a] \times [T]$$
$$= [LT^{-2}] \times [T]$$
$$= [LT^{-2+1}]$$
$$= [LT^{-1}]$$

Since, dimension of $[v] = [u] = [at]$, the given relation is correct.

iii) $T = 2\pi\sqrt{\frac{g}{l}}$

$$[T] = [T]$$

$$\left[2\pi\sqrt{\frac{g}{l}}\right] = [\sqrt{g/l}]$$
$$= [(g/l)^{1/2}]$$
$$= [(\cancel{L}T^{-2}/\cancel{L})^{1/2}]$$
$$= [(T^{-2})^{1/2}]$$
$$= [T^{-2\times\frac{1}{2}}]$$
$$= [T^{-1}]$$

Since, dimension of $[T] \neq 2\pi\sqrt{g/l}$, the given relation is not correct.

iv) $f = 2\pi\sqrt{g/l}$

$$[f] = \frac{[1]}{[T]}$$

$$[f] = [T^{-1}]$$

$[2\pi\sqrt{g/l}] = [\sqrt{g/l}]$

$= [(g/l)^{1/2}]$

$= [(\cancel{L}T^{-2}/\cancel{L})^{1/2}]$

$= [(T^{-2})^{1/2}]$

$= [(T^{-2\times\frac{1}{2}})]$

$= [T^{-1}]$

Since, dimension of $[f] = [2\pi\sqrt{g/l}]$, the given relation is correct.

v) $V = f\lambda$

$[V] = [LT^{-1}]$

$\cancel{[f] = \left[\frac{1}{T}\right] = [T^{-1}]}$

$[f\lambda] = [f] \times [\lambda]$

$= \left[\frac{1}{T}\right] \times [L]$

$= [T^{-1}] \times [L]$

$= [LT^{-1}]$

Since, dimension of $[V] = [f\lambda]$, the given relation is correct.

Check the correctness of the given relation. $t = 2\pi\sqrt{l/g}$

Given, $t = 2\pi\sqrt{l/g}$

$[t] = [T]$

$\left[2\pi\sqrt{l/g}\right] = \left[\sqrt{l/g}\right]$

$= \left[(l/g)^{\frac{1}{2}}\right]$

$= \left[\left(\cancel{L}/\cancel{L}T^{-2}\right)^{1/2}\right]$

$= \left[\left(1/T^{-2}\right)^{\frac{1}{2}}\right]$

$= \left[T^{2\times\frac{1}{2}}\right]$

$= [T]$

Since, the dimension ~~of~~ $[t] = \left[2\pi\sqrt{l/g}\right]$, the given relation is correct.

Scalar And Vectors

Scalar and vector quantities.

All the physical quantities used in physics can be grouped under two ~~q~~ categories.

i) Scalar quantities
ii) Vector quantities

Scalar Quantities

The physical ~~quantitie~~ quantities which are complete described by magnitude alone and are added according to the ordinary rules of algebra are called scalar quantities.

OR

Scalar quantities are the quantities having magnitude only and are added according to the ordinary rules of algebra.

E.g., Mass, distance, speed, volume, energy etc.

Vector Quantities

The physical quantities which need magnitude as well as direction for its complete description and are added according to the rules of vector addition are called vector quantities.

OR

Vector quantities are the quantities having both magnitude as well as direction and are added according to the rules of vector addition.

E.g., Weight, displacement, velocity, acceleration, force, pressure etc.

Representation of a Vectore

Suppose, A is a vectore quantity, then it is represent as $\vec{A}$ and is given by

$$\vec{A} = A\hat{A} \quad (\text{or}) \quad \vec{A} = |\vec{A}|\hat{A}$$

A (or) $|\vec{A}|$ is the magnitude of $\vec{A}$

$\hat{A}$ is called unit vectore gives direction of $\vec{A}$.

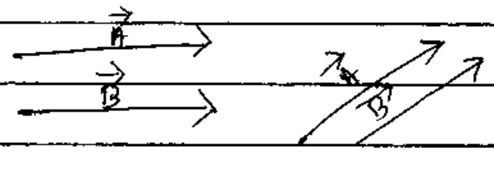
Types of Vectores

i) Null Vectore :- Null vectore is a vectore having zero magnitude and arbitrary direction.

ii) Unit Vectore :- Unit vectore is a vector having unit magnitude and is represented by $\hat{n}$ (or) $\hat{A}$ (or) $\hat{B}$ etc.

iii) Equal vector :- Two vectores are said to be equal if they have same magnitude as well as direction.

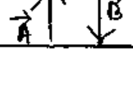
Let $\vec{A}$ and $\vec{B}$ are two equal vectors. Then $|\vec{A}| = |\vec{B}|$ (or) $A = B$



iv) Negative vectore :- A vectore is said to be negative of another vectore, if both have same magnitude and opposite in direct.

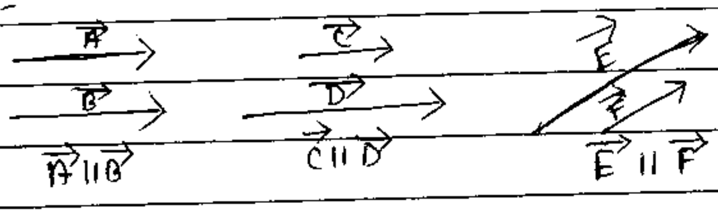
Let $\vec{A}$ & $\vec{B}$ are two vectores :-

$\vec{A}$ is said to be negative of $\vec{B}$ if $|\vec{A}| = |\vec{B}|$ but opposite in direction.



v) Parallel vector :- Two vectors are said to be parallel if they ~~tho~~ have same direction.

Example :-

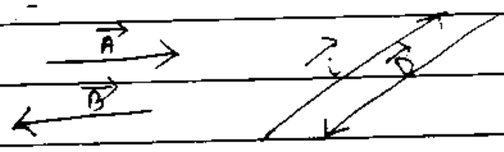


Symbolically, two parallel vectors are written as $\vec{A} \parallel \vec{B}$

Note :- The angle between two parallel vectors is zero.

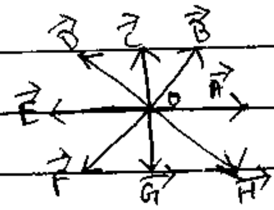
vi) Anti-Parallel vectors :- Two vectors are said to be anti-parallel if they are ~~in~~ opposite in direction.

Example :-



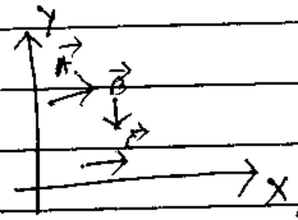
Note :- The angle between two anti-parallel vectors is 180°.

vii) Co-initial Vectors :- A number of vectors having a common inital point are called Co-inital vectors.



In the given figure, the vectors $\vec{A}, \vec{B}, \vec{C}, \vec{D}, \vec{E}, \vec{F}, \vec{G}$ and $\vec{H}$ all have the same initial point 'o'. So they are called co-inital vector.

viii) Co-Planer vector → Vectors lie in one plane are called Co-planer vector.



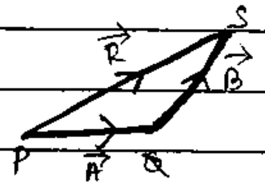
In the given figure, the vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ all are situa[ted] in the XY-plane. Therefore, these are called Co-planar vector.

Laws of Vector addition

There are two laws of vector addition
i) Triangle law of Vector addition.
ii) Parallelogram law of vector addition

Triangle law of vector addition.

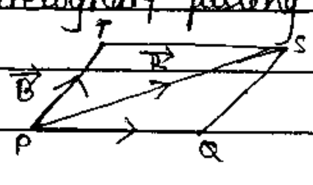
If two vectors are represented by two sides of a triangle take[n in] same order then ~~they are~~ their resultant vector is represent[ed] by the third side of the triangle take in the ~~sam~~ opposite order.



Consider two vectors $\vec{A}$ and $\vec{B}$ represented by two sides PQ and QS of the ΔPQS respectively, taken in the same order (anti-clockwise).
According to this law, their resultant vector $\vec{R}$ is represented b[y] the side PS, taken in opposite order (clockwise)
Here, $\vec{R} = \vec{A} + \vec{B}$

Parallelogram law of vector addition

Statement:- If two vectors acting simultaneously at a point are represented by two adject sides of a parallelogram drawn from a common point then their resultant is represented by the diagon[al] of the parallelogram passing through that point.



Consider two vectors $\vec{A}$ and $\vec{B}$ acting simultaneously at a point 'P' and are represented by the sides PQ and PT of the parallelogram PQST drawn from the same point 'P'.
According to this law their resultant $\vec{R}$ is represented by the diagonal PS passing through the same point 'P'.
Here,

$$\vec{A} + \vec{B} = \vec{R}$$

It can be proved mathematically,

$$R = \sqrt{a^2 + b^2 + 2ab\cos\theta}$$

OR

$$R^2 = a^2 + b^2 + 2ab\cos\theta$$

Where,
R = Magnitude of resultant $\vec{R}$
A = Magnitude of $\vec{A}$
B = Magnitude of $\vec{B}$
θ = angle between $\vec{A}$ and $\vec{B}$.

Problem - 1
Two vectors of magnitude 3 and 4 acting at an angle 90° to each other. Find the magnitude of Resultant vector.
Soln
Given $A = 3$
$B = 4$
$\theta = 90°$
$R = ?$

We have,

$$R = \sqrt{a^2 + b^2 + 2ab\cos\theta}$$

$R = \sqrt{3^2 + 4^2 + 2 \times 3 \times 4 \times \cos 90^\circ}$

$R = \sqrt{9 + 16 + 0}$

$R = \sqrt{25}$

$R = 5$

Problem - 2

Two forces 5 N and 4 N are acting ~~upon~~ at an angle 60° to each other. ~~Fid~~ Find the resultant force.

Solⁿ

Given = $F_1 = 5N$

$F_2 = 4N$

$\theta = 60^\circ$

$R = ?$

$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2 \cos\theta}$

$R = \sqrt{5^2 + 4^2 + 2 \times 5 \times 4 \times 1/2}$

$R = \sqrt{25 + 16 + 20}$

$F = \sqrt{61}\ N$

Problem 3

Resultant of two ~~equal~~ vectors is 149. Find the Magnitude of each vector if they have same Magnitude and acting at 90° to each other.

Problem 4

The resultant force of two forces is 25 N. If the angle between this two forces is 90° and one force is 3N. Find the Magnitude of the other force.

Sol^n of P-3

Given :- $R = 144$

$\theta = 90^\circ$

$A = ?$

$B = ?$

Here, $A = B$

So, $R^2 = A^2 + B^2 + 2AB\cos\theta$

$(144)^2 = A^2 + A^2 + 2 \times A \times A \cos 90^\circ$

$(144)^2 = 2A^2$

$144 \times 144 = 2A^2$

$144 \times 144 / 2 = A^2$

$$A = \sqrt{\frac{144 \times 144}{2}}$$

$$= \frac{144}{\sqrt{2}}$$

As, $A = B$, $B = \dfrac{144}{\sqrt{2}}$.

Sol^n of P-4

Given :- $F = 25$

$F_1 = 3N$

$\theta = 90^\circ$

$F_2 = ?$

$F = \sqrt{F_1^2 + F_2^2 + 2F_1 \times F_2 \cos 90^\circ}$

$25 = \sqrt{3^2 + F_2^2 + 2 \times 3 \times F_2 \times 0}$

$25 = \sqrt{9 + F_2^2 + 0}$

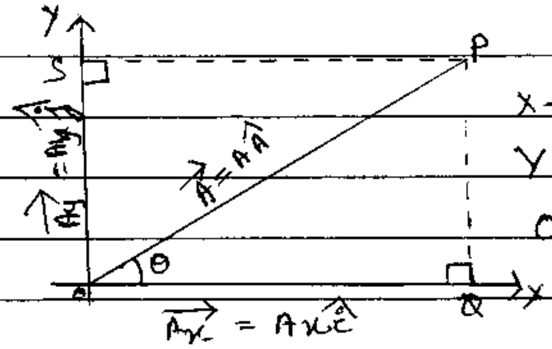
$(25)^2 = 9 + F_2^2$

$625 - 9 = F_2^2$

$F_2 = \sqrt{616}$ N

Resolution of a vector

Consider a vector a situated in the x - y plane as shown in the given figure.



X-axis (or) horizontal axis

Y-axis (or) Vertical axis

O → Origin

According to the resolution of the vector $\vec{A}$ can be written in terms of two components as follow $\vec{A} = \vec{A_x} + \vec{A_y}$ — (1)

Where $\vec{A_x}$ is the component of $\vec{A}$ along the x axis (or) horizontal axis and is called horizontal component.

A_y is the component of $\vec{A}$ along the y axis (or) Vertical axis and is called Vertical component.

So, we can write $\overrightarrow{A_x} = A_x\hat{i}$ —(2)

A_x → Magnitude of $\overrightarrow{A_x}$.

$\hat{i}$ → Unit vector along x-axis.

$\overrightarrow{A_y} = A_y\hat{j}$

A_y = Magnitude of $\overrightarrow{A_y}$

$\hat{j}$ = Unit vector along y-axis.

Putting equation (2) & (3) in equation (1) we get,

$\vec{A} = \cancel{A_x}\ A_x\hat{i} + A_y\hat{j}$ —(4)

In the Δ OPQ,

$\cos\theta = \frac{b}{h}$ or, $\cos\theta = \frac{OQ}{OP}$

or, $\cos\theta = \frac{A_x}{A}$

or, $A_x = A\cos\theta$ —— (5)

$\therefore$ The horizontal component of $\vec{A} = A\cos\theta$

In ΔOPQ,

$\sin\theta = \frac{p}{h}$

or, $\sin\theta = \frac{PQ}{OP}$

or, $\sin\theta = \frac{A_y}{A}$

or, $A_y = A\sin\theta$ —(6)

$\therefore$ The vertical component of $\vec{A} = A\sin\theta$.

Now, equation (4) can be written as $\vec{A} = A\cos\theta\,\hat{i} + A\sin\theta\,\hat{j}$ —(7)

- Problem-5.

Find the rectangular components of a force of 10 N acting at an angle 60° with the with the x axis in a xy plane

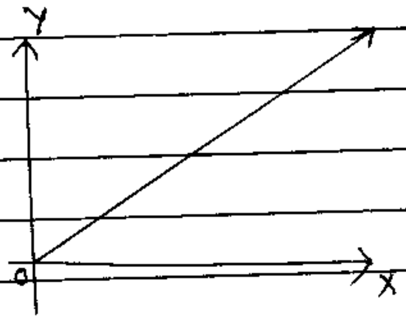
Solⁿ:-

Given :- $F = 10\,N$

$\theta = 60°$

$F_x = ?$

$F_y = ?$



$\therefore$ Horizontal component i.e., $F_x = F\cos\theta$

$= 10 \times \cos 60^\circ$

$= \cancel{10}^5 \times \frac{1}{\cancel{2}}$

$= 5\,N$

$\therefore$ Vertical component i.e., $F_y = F\sin\theta$

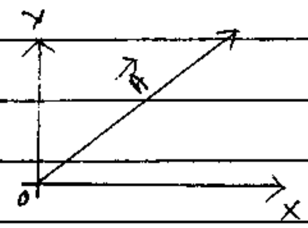
$= 10 \times \sin 60^\circ$

$= \cancel{10}^5 \times \frac{\sqrt{3}}{\cancel{2}}$

$= 5\sqrt{3}\,N.$

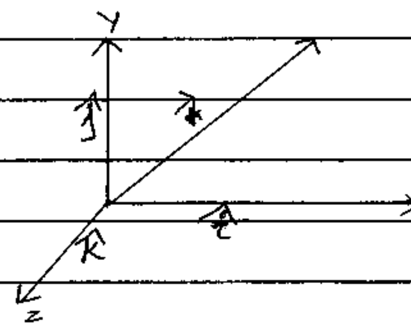
* Note :-

(2D) By vector resolution, $\vec{A} = A_x\hat{i} + A_y\hat{j}$



(3D) $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$

Where, $\hat{i}$, $\hat{j}$ and $\hat{k}$ are the unit vector along X, y, and Z axis respectively.



Vector Multiplication

There are two types of vector multiplication,

i) Dot product / scalar product

ii) Cross product / Vector product.

Dot Product

The dot product of two vectors $\vec{A}$ and $\vec{B}$ is defined as

$\vec{A}\cdot\vec{B} = AB\cos\theta$

Where, $A \rightarrow$ Magnitude of $\vec{A}$
$B \rightarrow$ Magnitude of $\vec{B}$
$\theta \rightarrow$ angle between $\vec{A}$ and $\vec{B}$.

Problem - 6
Find the dot product of two vectors having magnitude 3 and 4 and the angle between them is $60°$.

Soln
Given :- $A = 3$
$B = 4$
$\theta = 60°$
$\vec{A} \cdot \vec{B} = ?$

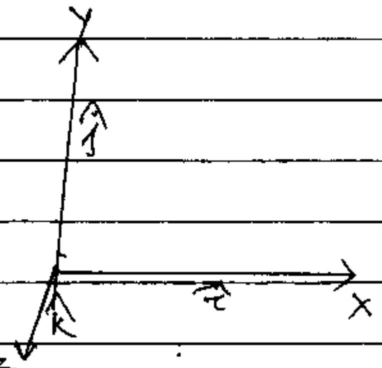
$\vec{A} \cdot \vec{B} = AB \cos 60°$
$= 3 \times 4 \times \cos 60°$
$= \cancel{12}^{6} \times \frac{1}{\cancel{2}}$

$= 6$

Note :-
The dot product is also called a scalar product.

Properties of Dot Product.
i) Dot product is commutative. $(\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A})$
ii) Dot product is distributive. $\{\vec{A} \cdot (\vec{B} + \vec{C}) = \vec{A} \cdot \vec{B} + \vec{A} \cdot \vec{C}\}$
iii) Dot product of two perpendicular vectors is zero.
Proof :-
Let $\vec{A}$ and $\vec{B}$ are two vectors and $\vec{A} \perp \vec{B}$
$\theta = 90°$.
$\vec{A} \cdot \vec{B} = AB \cos\theta$
or, $\vec{A} \cdot \vec{B} = \cancel{\cos} \, AB \cos 90°$

or, $\vec{A}\cdot\vec{B} = 0$



$\hat{i}\cdot\hat{j} = 0$
$\hat{j}\cdot\hat{k} = 0$
$\hat{k}\cdot\hat{i} = 0$

Where, $\hat{i}$, $\hat{j}$ and $\hat{k}$ are the unit vectors along X, Y and Z axis.

$|\hat{i}| = 1$
$|\hat{j}| = 1$
$|\hat{k}| = 1$

iv) Dot product of a unit vector with itself is one.

Let $\hat{A}$ is a unit vector.
$\therefore |\hat{A}| = 1$
$\hat{A}\cdot\hat{A} = |\hat{A}|\,|\hat{A}| \cos\theta$
Here, $\theta = 0^\circ$
$\hat{A}\,\hat{A} = 1 \times 1 \cos\theta$
$\hat{A}\,\hat{A} = 1$

$\hat{i}\cdot\hat{i} = 1$
$\hat{j}\,\hat{j} = 1$
$\hat{k}\,\hat{k} = 1$

v) Dot product in terms of rectangular components.
Let $\vec{A}$ and $\vec{B}$ are two vectors.
$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$
$\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$

$$\vec{A}.\vec{B} = (A_x\hat{i} + A_y\hat{j} + A_z\hat{k}).(B_x\hat{i} + B_y\hat{j} + B_z\hat{k})$$

$$= A_x\hat{i}.(B_x\hat{i} + B_y\hat{j} + B_z\hat{k}) + A_y\hat{j}(B_x\hat{i} + B_y\hat{j} + B_z\hat{k}) + A_z\hat{k}.(B_x\hat{i} + B_y\hat{j} + B_z\hat{k})$$

$$= A_xB_x\,\hat{i}.\hat{i} + \cancel{A_xB_y}\,A_xB_y\,\hat{i}.\hat{j} + A_xB_z\,\hat{i}.\hat{k} + A_yB_x\,\hat{j}.\hat{i} + A_yB_y\,\hat{j}.\hat{j} + A_yB_z\,\hat{j}.\hat{k} + A_zB_x\,\hat{k}.\hat{i} + A_zB_y\,\hat{k}.\hat{j} + A_zB_z\,\hat{k}.\hat{k}$$

$$= A_xB_x + 0 + 0 + 0 + A_yB_y + 0 + 0 + 0 + A_zB_z$$

$$\Rightarrow A_xB_x + A_yB_y + A_zB_z$$

Problem-7

If $\vec{A} = 2\hat{i} + \hat{j} + 3\hat{k}$ and $\vec{B} = \hat{i} + 3\hat{j} + 5\hat{k}$. Find $\vec{A}.\vec{B}$

Sol^n

- Given :- $\vec{A} = 2\hat{i} + \hat{j} + 3\hat{k}$

$A_x = 2$

$A_y = 1$

$A_z = 3$

$\vec{B} = \hat{i} + 3\hat{j} + 5\hat{k}$

$B_x = 1$

$B_y = 3$

$B_z = 5$

$$\vec{A}.\vec{B} = A_xB_x + A_yB_y + A_zB_z$$

$$= 2 \times 1 + 1 \times 3 + 3 \times 5$$

$$= 2 + 3 + 15$$

$$= 20$$

Problem - 8

If $\vec{A} = \hat{i} - 2\hat{j} + \hat{k}$ and $\vec{B} = 3\hat{i} + 4\hat{j} - 5\hat{k}$. Find $\vec{A}.\vec{B}$.

Sol^n

Given :- $\vec{A} = \hat{i} - 2\hat{j} + \hat{k}$

$A_x = 1,\ A_y = -2,\ A_z = 1$

$\vec{B} = 3\hat{i} + 4\hat{j} - 5\hat{k}$

$B_x = 3,\ B_y = 4,\ B_z = -5$

$$\vec{A}.\vec{B} = A_xB_x + A_yB_y + A_zB_z$$
$$= 1 \times 3 + (-2) \times 4 + 1 \times (-5)$$
$$= 3 - 8 - 5$$
$$= -10$$

Problem - 9

Find $\vec{A}.\vec{B}$ if $\vec{A} = \hat{i} - \hat{j}$ and $B = 2\hat{i} + 3\hat{j} - \hat{k}$

Sol^n

Given :- $\vec{A} = \hat{i} - \hat{j}$

$A_x = 1,\ A_y = -1,\ A_z = 0$

$\vec{B} = 2\hat{i} + 3\hat{j} - \hat{k}$

$B_x = 2,\ B_y = 3,\ B_z = -1$

$$\vec{A}.\vec{B} = A_xB_x + A_yB_y + A_zB_z$$
$$= 1 \times 2 + (-1) \times 3 + 0 \times -1$$
$$= 2 - 3 + 0$$
$$= -1$$

Problem - 10

Find dot product of following vectors :-

i) $\vec{A} = 2\hat{j} - \hat{k}$ and $\vec{B} = 2\hat{i} + \hat{j} + \hat{k}$

ii) $\vec{A} = \hat{i} - 2\hat{j} + 4\hat{k}$ and $\vec{B} = 2\hat{i}$

iii) $\vec{A} = 4\hat{k}$ and $\vec{B} = \hat{i} + 2\hat{j} - 3\hat{k}$

iv) $\vec{A} = 2\hat{i} + 3\hat{j} + 4\hat{k}$ and $\vec{B} = 2\hat{i} - 3\hat{j}$

Cross Product / vector product

If $\vec{A}$ and $\vec{B}$ are two vectors, the cross product of $\vec{A}$ and $\vec{B}$ is a vector and is defined as

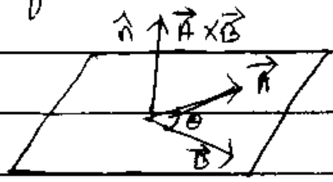
$\vec{A} \times \vec{B} = AB \sin\theta \, \hat{n}$

$A \rightarrow$ Magnitude of $\vec{A}$

$B \rightarrow$ Magnitude of $\vec{B}$

$\theta \rightarrow$ Angle between $\vec{A}$ and $\vec{B}$

$\hat{n}$ is a unit vector gives direction of $\vec{A} \times \vec{B}$



Note:-

i) The direction of $\vec{A} \times \vec{B}$ is along the perpendicular to the plain containing $\vec{A}$ and $\vec{B}$

ii) The magnitude of $\vec{A} \times \vec{B}$ is $|\vec{A} \times \vec{B}| = AB \sin\theta$

Problem - 11

Find the magnitude of cross product of two vectors having magnitudes ~~3x4~~ 3 and 4, acting at an angle 90° to each oth

Soln

Given :- $A = 3$

$B = 4$

$\theta = 90^\circ$

$|\vec{A} \times \vec{B}| = \cancel{A \times B \times}\ AB\sin\theta$

$= 3 \times 4 \times \sin 90^\circ$

$= 12 \times 1$

$= 12$

Properties of Cross product

i) Cross product is not commutative

i.e., $\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$

ii) Cross product is distributive.

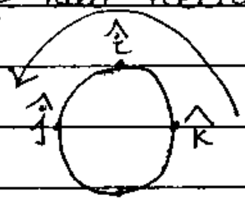
i.e., $\vec{A} \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C}$

iii) Cross product of two rectangular unit vector.

$\hat{i} \times \hat{j} = \hat{k}$

$\hat{j} \times \hat{k} = \hat{i}$

$\hat{k} \times \hat{i} = \hat{j}$

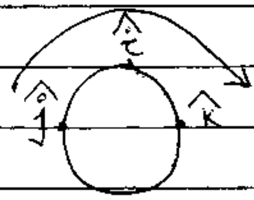


(Anti clockwise)

$\hat{i} \times \hat{k} = -\hat{j}$

$\hat{k} \times \hat{j} = -\hat{i}$

$\hat{j} \times \hat{i} = \hat{k}$



(clockwise)

iv) Cross product of a unit vector with itself.

OR

Cross product of two equal vectors.

If $\vec{A}$ and $\vec{B}$ are equal vectors

Here $\theta = 0°$

$\vec{A}$ →

$\vec{B}$ →

$\vec{A} \times \vec{B} = AB \sin\theta \, \hat{n}$

(or) $\vec{A} \times \vec{B} = AB \sin 0° \, \hat{n}$

$\vec{A} \times \vec{B} = 0$

$\hat{i} \times \hat{i} = 0$

$\hat{j} \times \hat{j} = 0$

$\hat{k} \times \hat{k} = 0$

Cross product in terms of rectangular component

Let $\vec{A}$ and $\vec{B}$ are two vectors.

We have,

$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$

$\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$

Problem 12.

Find $\vec{A} \times \vec{B}$ if $\vec{A} = \hat{i} + 2\hat{j} - 3\hat{k}$

$\vec{B} = 2\hat{i} - 3\hat{j}$

Sol^n

Given:- $\vec{A} = \hat{i} + 2\hat{j} - 3\hat{k}$

$\vec{B} = 2\hat{i} - 3\hat{j}$

$A_x = 1$, $A_y = 2$ $A_z = -3$

$B_x = 2$ $B_y = -3$ $B_z = 0$

Or $\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -3 \\ 2 & -3 & 0 \end{vmatrix}$

Or $\vec{A} \times \vec{B} = \hat{i}\begin{vmatrix} 2 & -3 \\ -3 & 0 \end{vmatrix} - \hat{j}\begin{vmatrix} 1 & -3 \\ 2 & 0 \end{vmatrix} + \hat{k}\begin{vmatrix} 1 & 2 \\ 2 & -3 \end{vmatrix}$

$= \hat{i}(0-9) - \hat{j}\,[0-(-6)] + \hat{k}(-3-4)$

$\vec{A} \times \vec{B} = -9\hat{i} - 6\hat{j} - 7\hat{k}$

Problem -13

If $\vec{A} = \hat{i} + \hat{j} - 2\hat{k}$

$\vec{B} = 3\hat{i} - 4\hat{k}$

Find $\vec{A} \times \vec{B}$

Sol^n

Given :- $\vec{A} = \hat{i} + \hat{j} - 2\hat{k}$

$A_x = 1 \quad A_y = 1 \quad A_z = -2$

$\vec{B} = 3\hat{i} - 4\hat{k}$

$B_x = 3 \quad B_y = 0 \quad B_z = -4$

or $\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -2 \\ 3 & 0 & -4 \end{vmatrix}$

or $\vec{A} \times \vec{B} = \hat{i} \begin{vmatrix} 1 & -2 \\ 0 & -4 \end{vmatrix} - \hat{j} \begin{vmatrix} 1 & -2 \\ 3 & -4 \end{vmatrix} + \hat{k} \begin{vmatrix} 1 & 1 \\ 3 & 0 \end{vmatrix}$

$= \hat{i}(-4) - \hat{j}(-4 - (-6)) + \hat{k}(0 - 3)$

$= -4\hat{i} - 2\hat{j} - 3\hat{k}$

Problem - 14

Find the cross product of vectors given in Problem - 10.

Note.

If $\vec{A} = A_x\hat{i} + A_y\hat{j}$

$A_x \rightarrow$ Horizontal component (H.C)

$A_y \rightarrow$ Vertical component (V.C)

$A^2 = A_x^2 + A_y^2$

$A^2 = (H.C)^2 + (V.C)^2$

ii) If $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$

$A^2 = A_x^2 + A_y^2 + A_z^2$

$A = \sqrt{A_x^2 + A_y^2 + A_z^2}$

Solⁿ of Problem - 12

i) $\vec{A} = 2\hat{j} - \hat{k}$ and $\vec{B} = 2\hat{i} + \hat{j} + \hat{k}$

Given :- $\vec{A} = 2\hat{j} - \hat{k}$

$A_x = 0$, $A_y = 2$, $A_z = -1$

$\vec{B} = 2\hat{i} + \hat{j} + \hat{k}$

$B_x = 2$, $B_y = 1$, $B_z = 1$

$\vec{A}.\vec{B} = A_x B_x + A_y B_y + A_z B_z$

$= (0 \times 2) + (2 \times 1) + (-1 \times 1)$

$= 0 + 2 - 1$

$= 1$

ii) $\vec{A} = \hat{i} - 2\hat{j} + 4\hat{k}$ and $\vec{B} = 2\hat{i}$

Given :- $\vec{A} = \hat{i} - 2\hat{j} + 4\hat{k}$

$A_x = 1$, $A_y = -2$, $A_z = 4$

$\vec{B} = 2\hat{i}$

$B_x = 2$, $B_y = 0$, $B_z = 0$

$\vec{A}.\vec{B} = A_x B_x + A_y B_y + A_z B_z$

$= (1 \times 2) + (-2 \times 0) + (4 \times 0)$

$= 2 + 0 + 0$

$= 2$

iii) $\vec{A} = 4\hat{k}$ and $\vec{B} = \hat{i} + 2\hat{j} - 3\hat{k}$

Given :- $\vec{A} = 4\hat{k}$

$A_x = 0$, $A_y = 0$, $A_z = 4$

$\vec{B} = \hat{i} + 2\hat{j} - 3\hat{k}$

$B_x = 1$, $B_y = 2$, $B_z = -3$

$$\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z$$
$$= (0\times 1) + (0\times 2) + (4\times -3)$$
$$= 0 + 0 + (-12)$$
$$= -12$$

iv) $\vec{A} = 2\hat{i} + 3\hat{j} + 4\hat{k}$ and $\vec{B} = 2\hat{i} - 3\hat{j}$

Given :- $\vec{A} = 2\hat{i} + 3\hat{j} + 4\hat{k}$

$A_x = 2,\ A_y = 3,\ A_z = 4$

$\vec{B} = 2\hat{i} - 3\hat{j}$

$B_x = 2,\ B_y = -3,\ B_z = 0$

$$\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z$$
$$= (2\times 2) + [3\times(-3)] + (4\times 0)$$
$$= 4 + (-9) + 0$$
$$= -5$$

Sol^n of Problem - 14

i) $\vec{A} = 2\hat{j} - \hat{k}$ and $\vec{B} = 2\hat{i} + \hat{j} + \hat{k}$

Given :- $\vec{A} = 2\hat{j} - \hat{k}$

$A_x = 0,\ A_y = 2,\ A_z = -1$

$\vec{B} = 2\hat{i} + \hat{j} + \hat{k}$

$B_x = 2,\ B_y = 1,\ B_z = 1$

$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 2 & -1 \\ 2 & 1 & 1 \end{vmatrix}$$

$$\vec{A}\times\vec{B} = \hat{i}\begin{vmatrix} 2 & -1 \\ 1 & 1 \end{vmatrix} - \hat{j}\begin{vmatrix} 0 & -1 \\ 2 & 1 \end{vmatrix} + \hat{k}\begin{vmatrix} 0 & 2 \\ 2 & 1 \end{vmatrix}$$
$$= \hat{i}[2-(-1)] - \hat{j}[0-(-2)] + \hat{k}(0-4)$$

$= 3\hat{i} - 2\hat{j} - 4\hat{k}$

ii) $\vec{A} = \hat{i} - 2\hat{j} + 4\hat{k}$ and $\vec{B} = 2\hat{i}$

Given :- $\vec{A} = \hat{i} - 2\hat{j} + 4\hat{k}$

$A_x = 1$, $A_y = -2$, $A_z = 4$

$\vec{B} = 2\hat{i}$

$B_x = 2$, $B_y = 0$, $B_z = 0$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 4 \\ 2 & 0 & 0 \end{vmatrix}$$

$$\vec{A} \times \vec{B} = \hat{i} \begin{vmatrix} -2 & 4 \\ 0 & 0 \end{vmatrix} - \hat{j} \begin{vmatrix} 1 & 4 \\ 2 & 0 \end{vmatrix} + \hat{k} \begin{vmatrix} 1 & -2 \\ 2 & 0 \end{vmatrix}$$

$= \hat{i}\,(0 - 0) - \hat{j}\,(0 - 8) + \hat{k}\,\{0 - (-4)\}$

$= 8\hat{j} + 4\hat{k}$

iii) $\vec{A} = 4\hat{k}$ and $\vec{B} = \hat{i} + 2\hat{j} - 3\hat{k}$

Given :- $\vec{A} = 4\hat{k}$

$A_x = 0$, $A_y = 0$, $A_z = 4$

$\vec{B} = \hat{i} + 2\hat{j} - 3\hat{k}$

$B_x = 1$, $B_y = 2$, $B_z = -3$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 4 \\ 1 & 2 & -3 \end{vmatrix}$$

$$\vec{A} \times \vec{B} = \hat{i} \begin{vmatrix} 0 & 4 \\ 2 & -3 \end{vmatrix} - \hat{j} \begin{vmatrix} 0 & 4 \\ 1 & -3 \end{vmatrix} + \hat{k} \begin{vmatrix} 0 & 0 \\ 1 & 2 \end{vmatrix}$$

$\cancel{\hat{i}}\ \vec{A}\times\vec{B} = \hat{i}\,(0-8) - \hat{j}\,(0-4) + \hat{k}\,(0-0)$

$= \cancel{-8\hat{i}+4\hat{j}}\ \ -8\hat{i}+4\hat{j}$

iv) $\vec{A} = 2\hat{i} + 3\hat{j} + 4\hat{k}$ and $\vec{B} = 2\hat{i} - 3\hat{j}$

Given :- $\vec{A} = 2\hat{i} + 3\hat{j} + 4\hat{k}$

$A_x = 2,\ A_y = 3,\ A_z = 4$

$\vec{B} = 2\hat{i} - 3\hat{j}$

$B_x = 2,\ B_y = -3,\ B_z = 0$

$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 2 & -3 & 0 \end{vmatrix}$$

$$\vec{A}\times\vec{B} = \hat{i}\begin{vmatrix} 3 & 4 \\ -3 & 0 \end{vmatrix} - \hat{j}\begin{vmatrix} 2 & 4 \\ 2 & 0 \end{vmatrix} + \hat{k}\begin{vmatrix} 2 & 3 \\ 2 & -3 \end{vmatrix}$$

$= \hat{i}\,(0-(-12)) - \hat{j}\,(0-8) + \hat{k}\,(-6-6)$

$= \cancel{12\hat{i}+8\hat{j}-k}\ \ 12\hat{i} + 8\hat{j} - 12\hat{k}$

Problem-15

One of the rectangular components of a force of 65 N is 25N. Find the other component.

Given :- $F = 65\,N$

$F_x = 25\,N$

$F_y = ?$

$F_y = \sqrt{(F)^2 - (F_x)^2}$

$= \sqrt{(65)^2 - (25)^2}$

$= \sqrt{4225 - 625}$

$= \sqrt{3600}$

$= 60\,N.$

Problem -16.

Find the rectangular components of a velocity of 8 m/s, where one of the components makes an angle of 30° with the resultant.

Given :- $V = 8\,m/s$

$\theta = 30^\circ$

We know, Vertical component $= V \sin\theta$

$= 8 \times \sin 30^\circ$

$= 8 \times \frac{\sqrt{3}}{2}$

$= 4\sqrt{3}\ m/s.$

∴ Horizontal component $= V\cos\theta$

$= 8 \times \cos 30^\circ$

$= 8 \times \frac{1}{2}$

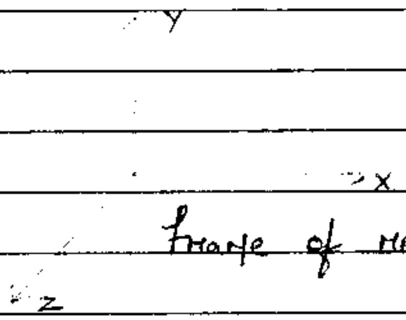
$= 4\ m/s.$

Kinematics

Concept of Rest and Motion

The term rest and motion are not absolute and are relative.
So, the defination of rest (or) motion needs a frame of reference.

A convinient way to fix up a frame of reference is to choose three mutual perependicular axis and name them X, Y, Z.



Frame of reference.

The coordinates (x, y, z) gives the ~~positive~~ position of body in that frame of reference.
Add a clock to the frame of reference to measure time.

Defination of Rest

If all the three coordinates (x, y, z) of a body don't change with time then the body is said to at rest with respect to the frame of reference.

Defination of Motion

If any one or more coordinates (x, y, z) of a body changes with time then the body is said to be in motion with respect to the frame of reference.

Distance

Defination:- The length of a path taken by a body is called distance.

Suppose, body takes path - 4 (P-4) to travel from initial point to final point B (as shown in the fig - 1. The length of the path - 4 is

Called the distance travelled by the body.

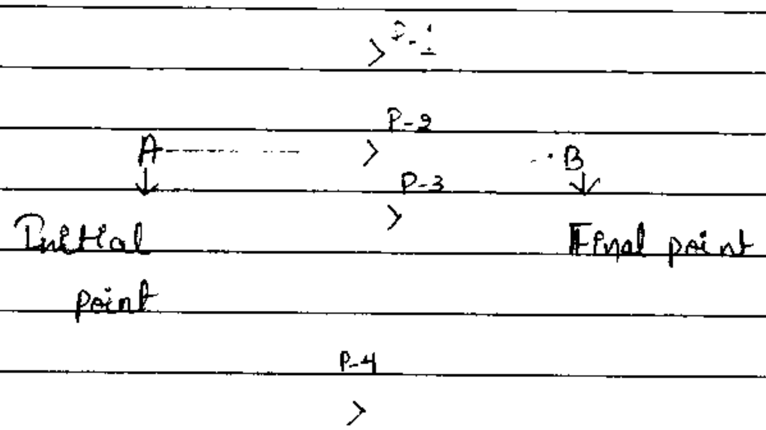


fig :- 1

→ It is a scalar quantity.
→ It is denoted by S.
→ Its S.I unit is meter (m)
→ Its dimensional formula is [S] = [L]

Displacement

It is a vector quantity whose magnitude is the length of the shortest path between initial point and final point and its direction is always from initial point to the final point.

→ It is a vector quantity.
→ It is denoted by $\vec{S}$.
→ Its S.I unit is meter. (m)
→ Its DF is $[\vec{S}] = [L]$

Note :-

When the initial point and final point of a body is the same the displacement of a body is zero.

Problem - 1

A person covers a circular path of radius 21m in his morning walk. Find its distance and displacement.

Given :-

Radius = 21m.

Distance travelled by the body in circular path = $2\pi r$

$= 2 \times \frac{22}{\cancel{7}} \times \cancel{21}^{3}$

$= 132$ m.

Displacement = 0

Speed

The speed of a body is equal to the distance travell divided by the time taken to cover the distance.

i.e., Speed = $\frac{\text{Distance}}{\text{Time}}$

Example :- Distance = 200m

Time = 10 sec.

Speed = ?

Speed = $\frac{20\cancel{0}}{1\cancel{0}}$

= 20m/s.

→ It is a scalar quantity

→ It is denoted by ~~(u)~~ u (or) v.

→ Its S.I unit is m/s, meter/second, ms^{-1}.

→ Its DF is $[LT^{-1}]$.

Speed

↓ Average Speed

↓ Instantaneous speed (or) Speed

Average Speed

Average speed of a body in a time interval may be defined as the total distance travelled by the body divided by the time interval.

i.e, average speed = $\dfrac{\text{Total distance}}{\text{Total Time}}$

E.g.,

Total distance = 400 + 400

= 800 m

Total time = 20 + 40

= 60 sec.

Average Speed = $\dfrac{\cancel{800}^{40}}{\cancel{60}_{3}}$

$= \dfrac{40}{3}$

= 13.33 m/s.

~~Intan~~ Instantneous Speed / Speed

Instantneous Speed (or) speed of a body deals with the time interval is very small i.e, $\Delta t \to 0$

Instantneous Speed (or) speed

Deffnation :-

Speed of a body may be defined as rate of change of distance with time.

Mathematically,

$$V = \frac{ds}{dt}$$

Velocity

The Velocity of a body is the displacement divided by the time ta
to cover the displacement.

i.e, $\text{Velocity} = \frac{\text{Displacement}}{\text{Time}}$

For example :- $S = 100\,m$

$t = 10\,s$

$\text{Velocity} = \frac{100}{10}$

$= 10\,m/s.$

→ It is a vector quantity.
→ Its S.I unit is m/s.
→ It is denoted by $\vec{u}$ (or) $\vec{v}$.
→ Its DF is $[V] = [L T^{-1}]$.

Velocity
↓ Average Velocity
↓ Instantaneous velocity (or) Velocity

Average velocity is total displacement upon total time taken by an object to travel.

Velocity :- It deals with when time interval Δt is very small i.e.
$\Delta t \to 0$ and $\vec{V} = \frac{d\vec{s}}{dt}$

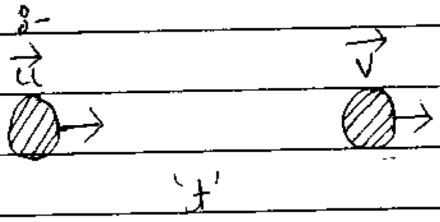
Rate of change of displacement with time.

Acceleration

Acceleration of a body is the change in velocity of the body with time

i.e, acceleration = $\frac{\text{Change in velocity}}{\text{Time}}$

For example :-



$\vec{u}$ = initial velocity

$\vec{v}$ = final velocity / velocity of the body after time 't'.

∴ Acceleration of the body is given by $\vec{a} = \frac{\vec{v} - \vec{u}}{t}$

→ It is denoted by $\vec{a}$.

→ It is a vector quantity.

→ Its ~~is~~ S.I unit is m/s^2 (or) ms^{-2}.

→ Its DF is $[a] = [LT^{-2}]$

The acceleration of a body may also defined as rate of change of velocity.

i.e, $\vec{a} = \frac{d\vec{v}}{dt}$

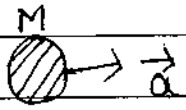
When the velocity of a body increases it is called acceleration and when the velocity of a body decreases it is called retardation.

Force

Force on a body may be defined as the product of the mass of the body and acceleration of the body

If M = Mass of the body
a = acceleration of the body, then

Force = $\vec{F} = M\vec{a}$



→ $\vec{F}$

F Direction

Magnitude $F = Ma$ along direction of acceleration

→ It is a vector quantity
→ Its S.I unit is Newton (N).
→ Its dimensional formula (DF) is $[MLT^{-2}]$.

P-2 If a body of mass 5kg is moving with an acceleration of $10 m/s^2$
Find the force on the body.

Given :- Mass = 5 kg
Acceleration = $10 m/s^2$.
Force = ?

Force = $M \times a$
$= 5 \times 10$
$= 50 N$

P-3 If the acceleration of a body is $10 m/s^2$. When a force of 50N is applied, then find the mass.

Given :- Force = 50 N

acceleration = 10 m/s²

$$\text{Mass} = \frac{\text{Force}}{\text{acceleration}}$$

$$= \frac{5\cancel{0}}{1\cancel{0}}$$

$$= 5\,kg.$$

P-4 The velocity of a body changes from 5 m/s to 10 m/s in 5 sec. Find the acceleration in the body.

Given :- $u = 5\,m/s$

$V = 10\,m/s$

$t = 5\,sec.$

$$a = \frac{v-u}{t}$$

$$= \frac{10-5}{5}$$

$$= \frac{5}{5}$$

$$= 1\,m/s^2.$$

Motion in One dimension are One dimensional Motion.

Motion of a body along a straight line → ↑ ↓ ←.

The one dimensional motion is governed by three equations.

$V = u + at$ —(1)

$S = ut + \frac{1}{2}at^2$ —(2)

$V^2 = u^2 + 2as$ —(3)

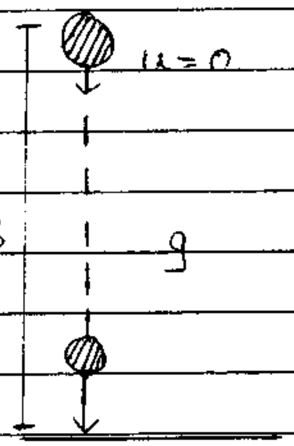
$u \rightarrow$ Magnitude of initial velocity.
$V \rightarrow$ Magnitude of final velocity.
$t \rightarrow$ Time
$a \rightarrow$ acceleration
$S \rightarrow$ Magnitude of displacement.

Equations Of Motion Under Gravity

Downward Motion

Consider a body is released from a certain height in the downward direction towards the earth.

Case - 1



$g\downarrow\ V\downarrow\ S\downarrow$

Here, $a = g$, $u = 0$, $s = h$

$\therefore$ Equations of motion under gravity for the case are :-

i) $V = 0 + gt$

$v = gt$

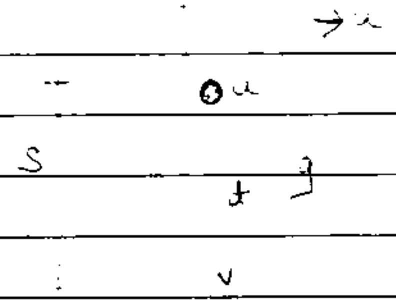
ii) $h = 0 \times t + \frac{1}{2}gt^2$

$h = \frac{1}{2}gt^2$

iii) $V^2 = 0^2 + 2gh$

$V^2 = 2gh$

Case 2 :-



$u\downarrow$ $v\downarrow$ $g\downarrow$ $S\downarrow$
$+$ $+$ $+$ $+$

Here, $a = g$, $S = h$

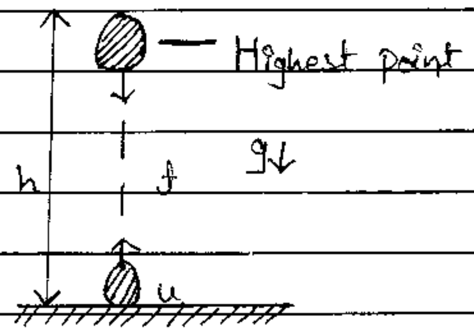
The equation for the motion for this case are :-

i) $V = u + gt$

ii) $h = u + \frac{1}{2} gt^2$

iii) $v^2 = u^2 + 2gh$

Upward Motion



Consider a body is thrown upward with initial velocity u.
Here, $V = 0$, $a = -g$, $S = h$

The equation of motion for upward direction are as follow :-

i) $0 = u - gt$

(Or)

$u = gt$

ii) $h = ut + \frac{1}{2}(-g)t$

$h = ut - \frac{1}{2}gt^2$

iii) $0^2 = u^2 + 2(-g)h$

(or) $u = \sqrt{2gh}$

$u^2 = 2gh$

Circular Motions

In order to define motion we need a fixed point or reference point with respect to which the motion of a body can be defined.

When the distance between the fixed point and a body changes with time then the body is said to be in motion.

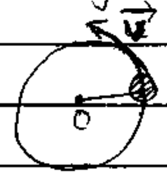
However, there is a type of motion in which the distance between fixed point and the body doesn't change with time and the body is said to be under circular motion.

Definition :-

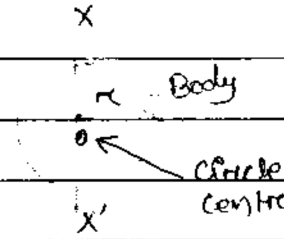
The motion of a body is said to be circular if the body moves in such a way that the distance between a fixed point and the body remains the same throughout the motion.

Direction of motion of a body moving under circular motion ~~of body~~ at any instant.

The direction of motion of a body at any instant is always along the tangent to the circle at the point.



Terms related to circular Motion



Consider a body is moving under circular motion.

* Circle :- The path covered by the body under circular motion is called circle (or) circular path.

* Centre :- The fixed point of the circular motion is called centre of the circle.

* Radius :- The distance between the centre and the body is called radius of the circle. It is denoted by 'r'.

* Axis of rotation :- It is a straight line passing through the centre of the circle and perpendicular to the plane of the circle.
  Here, $XX' \rightarrow$ axis of rotation.

* Time period :- It is the time taken by a body to cover one complete rotation along circular path.
  It is denoted by T.

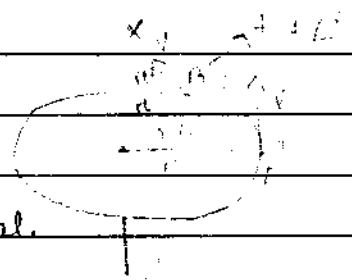
* Frequency :- It is the number of ~~complete~~ complete rotations done by a body per second.
  It is denoted by f (or) n (or) ν (neu)

Relation between T and f.

$$\text{Time period} = \frac{1}{\text{frequency}} \quad (\text{or}) \quad T = \frac{1}{f} \quad (\text{or}) \quad f = \frac{1}{T}$$

Angular Displacement

Angular displacement of a body undergoing circular motion is defined as the angle covered by the body in a given time interval.



In the given figure, the body covers an angle $\Delta\theta$ in the time interval Δt.

$\therefore$ Angular displacement $= \Delta\theta$.

→ It is a vector quantity.

→ It is denoted by '$\vec{\theta}$'.

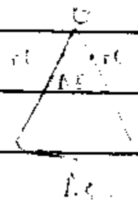
→ Its S.I unit is radian (rad).

~~The sense of angular displacement can be determined~~

→ The direction of the angular displacement is along the axis of rotation and its sense can be determined by using right hand thumb rule.

Relation between linear displacement of angular displacement.

Let ΔS and $\Delta\theta$ be the linear displacement and angular displacement of a body moving in a circular path.



Scalar form

$$\Delta S = r\Delta\theta$$

Vector form

$$\vec{\Delta S} = \vec{\Delta\theta} \times \vec{r}$$

Angular Velocity

Defination :- The angular velocity of a body under going circular motion is defined as the rate of change of angular displacement with time.

Mathematically,

$$\vec{\omega} = \frac{d\vec{\theta}}{dt} \quad (\omega \to \text{Omega}).$$

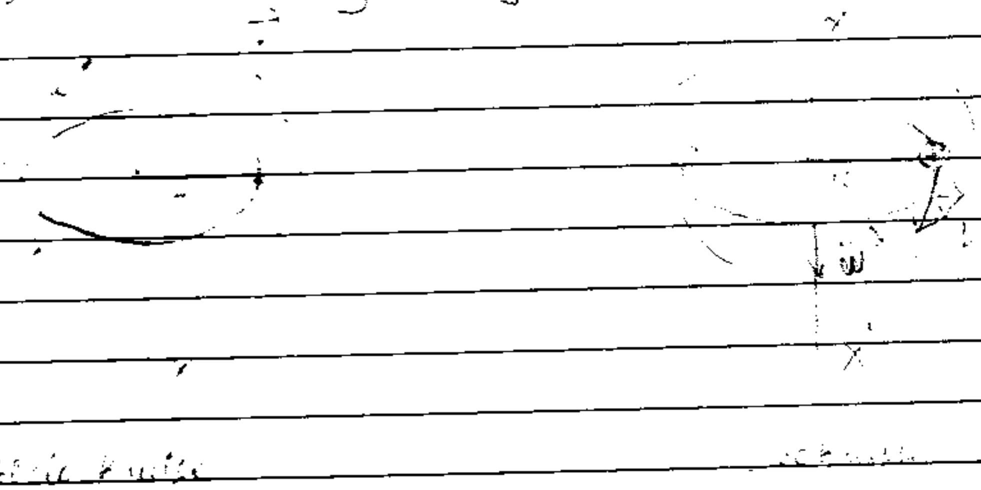
$\vec{\omega}$ → Angular velocity.

→ It is a vector quantity.

It is denoted by $\vec{\omega}$.

Its S.I unit is $\frac{\text{radian}}{\text{second}}$ (or) $\frac{\text{rad}}{\text{sec}}$

→ Its direction is along the axis of rotation and its sense can be determined by using RHTR



Relation between linear velocity and angular velocity.

Let $\vec{V}$ → linear velocity and

$\vec{\omega}$ → Angular velocity of a body moving in a circular path of radius r.

Scalar form

$$V = r\omega$$

Vector form

$$\vec{V} = \vec{\omega} \times \vec{r}$$

Relation between time period and angular velocity.

Let T be the time period and ω be the angular displacement of a body

For a complete rotation
Angular displacement, $\theta = 2\pi$ radian

$\therefore \omega = \frac{\theta}{T}$

(or) $\omega = \frac{2\pi}{T}$

Problem - 1
Calculate the angular velocity of
i) Second's hand
ii) Minute's hand
iii) hour's hand of a clock.

Problem - 2
A car goes round a curve of radius 8m with a velocity of 54 km/h
What is its angular velocity.

Soln - 1
Given :- i) Second hand
$\theta = 2\pi$ radian.
$T = 60s$

$\omega = \theta/T = 2\pi/60 = \pi/30$ rad/s.

ii) Minute hand : $\theta = 2\pi$ radian, $T = 3600s$.

$\omega = \theta/T = 2\pi/3600 = \pi/1800$ rad/s.

iii) Hour hand
$\theta = 2\pi$ radian, $T = 43200s$

$\omega = \theta/T = 2\pi/43200 = \pi/21600$ rad/s.

Sol^n

Given :- radius = 8 m

velocity = 54 km/h = 54 × 1000/3600 = 15 m/s.

Angular velocity = ?

~~Angular velocity =~~

We have, $V = r\omega$

$\Rightarrow \omega = r/v$

$\Rightarrow \omega = \frac{V}{r} = \frac{15}{8} = 1.87$ rad/s.

Angular Acceleration

Defination

The acceleration of a $\vec{V}$ body undergoing circular motion is defined as the rate of change of angular velocity with time.

Mathematically,

$$\alpha = \frac{d\omega}{dt}$$

Where $\alpha \rightarrow$ angular acceleration

→ It is a vector quantity.

→ It is denoted by α

→ Its S.I unit is rad/s^2

→ Its direction is along the axis of rotation

Relation between linear acceleration and angular acceleration

Let a be the linear acceleration and α be the angular acceleration of a body moving in a circular path of radius 'r'.

Scalar form

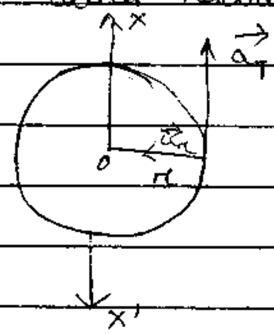
$$a = r\alpha$$

Vector form

$$\vec{a} = \vec{r} \times \vec{\alpha} + \vec{\omega} \times \vec{v}$$

(or) $\vec{a} = \vec{a}_T + \vec{a}_r$

where $\vec{a}_T = \vec{\alpha} \times \vec{r}$ and is called tangential component.
$\vec{a}_r = \vec{\omega} \times \vec{v}$ and is called radial component.



Problem - 3
The angular velocity of a body changes 4π rad/s to 5π rad/s in 10 sec. What is its angular acceleration.

Given :-

$\omega_i = 4\pi$ rad/s
$\omega_f = 5\pi$ rad/s
$t = 10$ sec
$\alpha = ?$

We have $\alpha = \dfrac{\omega_f - \omega_i}{t} = \dfrac{5\pi - 4\pi}{10} = \dfrac{\pi}{10}$ rad/s^2.

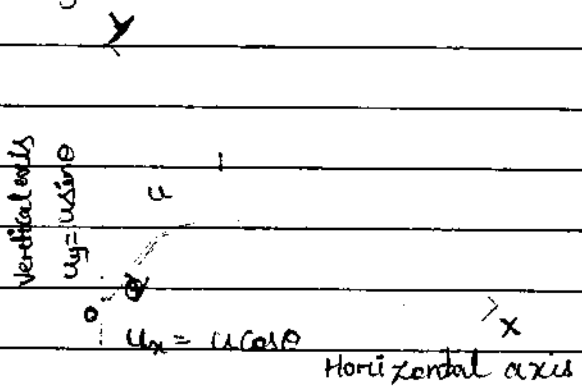
Projectile Motion

Definition :- Projectile motion is a two dimensional motion in which a body moves under the effect of gravity (gravitational force) only.
Example :- i) The motion of a bullet fired from a gun.
ii) Falling of a fruit from a tree.

iii) Falling of a bag from a helicopter.
These are the examples of projectile motion.

A projectile is fired with velocity u, making an angle θ with the horizontal. Derive expressions for.

i) Equations of trajectory.
ii) Maximum height
iii) Time of ascent
iv) Time of desent
v) Time of flight
vi) Horizontal range



$y = mx + c$ → equation of straight line.
$x^2 + y^2 = r^2$ → equation of circle
$y^2 = 4ax + b$ → equation of parabola
$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ → equation of ellipse.

Let a projectile is fired with velocity $\vec{u}$ at an angle θ with the horizontal, the initial velocity u can be resolved into two component as follow.
$u_x = u\cos\theta$ —(1) and is horizontal component along x-axis.
$u_y = u\sin\theta$ —(2) and is vertical component along y-axis.

In projectile motion the horizontal component of acceleration of 'a' is zero (i.e.) $a_x = 0$ —(3)

and the vertical component of acceleration a is $-g$ i.e., $a_y = -g$ — (4)

i) Expression for equation of trajectory

We have the relation $S = ut + \frac{1}{2}at^2$ —(1)

Consider the motion of the projectile in the horizontal direction only, the equation -(1) can be written as $S_x = u_x t + \frac{1}{2} a_x t^2$

Here,

$S_x = x$, ~~ax =~~ $a_x = 0$, $u_x = u\cos\theta$

$\therefore x = u\cos\theta . t + \frac{1}{2} \times 0 \times t^2$

$x = u\cos\theta\, t + 0$

(or) $x = u\cos\theta\, t$

(or) $t = x / u\cos\theta$ — (2)

Considering the motion of the projectile in vertical direction only, the equation (1) can be written as.

$S_y = u_y t + \frac{1}{2} a_y t^2$

Here, $S_y = y$, $u_y = u\sin\theta$, $a_y = -g$

$\therefore y = u\sin\theta . t + \frac{1}{2}(-g) t^2$

(or) $y = u\sin\theta\, t - \frac{1}{2} g t^2$ — (3)

Substituting $t = x / u\cos\theta$ in equation —(3) we get

~~y = usinθ x u/sin~~

$y = u\sin\theta \times u / u\cos\theta - \frac{1}{2} g \left(x / u\cos\theta\right)^2$

(or) $y = (\sin\theta / \cos\theta) x - \frac{1}{2} g\, x^2 / u^2\cos^2\theta$

(or) $y = \tan\theta\, x - \left(g / 2u^2\cos^2\theta\right) x^2$

let $\tan\theta = a$ and $g / 2u^2\cos^2\theta = b$

$\therefore y = ax - bx^2$ — (4)

equation (4) shows the nature of the trajectory is prabola.

ii) Expression for Maximum height

Maximum height of a projectile is the maximum distance travelled by the projectile in vertical direction.

It is denoted by H.

It is noted that the verticle component of the velocity of the body at the highest point A is zero i.e, $V_y = 0$.

We have the relation $v^2 = u^2 + 2as$ —(1)

Considering motion of the projectile in vertical direction only the equation —(1) becomes $V_y^2 + u_y^2 + 2a_y S_y$

Here, $V_y = 0$, $u_y = u\sin\theta$, $a_y = -g$, $S_y = H$

$\therefore\ 0^2 = (u\sin\theta)^2 + 2(-g)H$

(or) $0 = u^2\sin^2\theta - 2gH$

(or) $2gH = u^2\sin^2\theta$

(or) $H = \dfrac{u^2\sin^2\theta}{2g}$ — (2)

Which is the required expression.

iii) Expression for time of ascent

Time of ascent of a projectile is the time taken by the projectile to reach the highest point.

It is denoted by t_a.

We have the relation $V = u + at$ —(1)

Considering the motion of the projectile in vertical direction only the equation becomes $V_y = u_y + a_y t$

Here, $V_y = 0$, $u_y = u\sin\theta$, $a_y = -g$, $t = t_a$

$\therefore\ 0 = u\sin\theta + (-g)t_a$

(or), $0 = u\sin\theta - gt_a$

(or) $gt_a = u\sin\theta$

$$t_a = \frac{u\sin\theta}{g} \quad \text{---(2)}$$

iv) Expression for time of descent.

The time of descent of a projectile may be defined as the time taken by the projectile to reach the ground level from the highest point.

It is denoted by t_d.

It is noted that time of ascent is equal to the time of descent.

i.e., $t_d = t_a$

$$\Rightarrow t_d = \frac{u\sin\theta}{g}$$

v) Expression for time of flight.

The time of flight of a projectile may be defined as the total time taken by the projectile to reach the same level of projection again.

It is denoted by T.

$\therefore\ T = t_a + t_d$

$$\text{(or)}\ T = \frac{u\sin\theta}{g} + \frac{u\sin\theta}{g}$$

$$\text{(or)}\ T = \frac{2u\sin\theta}{g}$$, which is the required expression

vi) Expression for horizontal range

Horizontal range of a projectile may be defined as the distance covered by the projectile in horizontal direction in the time interval $\frac{2u\sin\theta}{g}$.

It is denoted by R.

We have the relation $S = ut + \frac{1}{2}at^2$ —(1)

Considering the motion of the projectile in horizontal direction only, the equation —(1) becomes $S_x = u_x t + \frac{1}{2}a_x$

Here, $S_x = R$

$u_x = u\cos\theta$

$t = T = \frac{2u\sin\theta}{g}$

$a_x = 0$

$$\therefore R = u\cos\theta \times \frac{2u\sin\theta}{g} + \frac{1}{2} \times 0 \times \left(\frac{2u\sin\theta}{g}\right)^2$$

$$\text{(or) } R = \frac{2u^2\sin\theta\cos\theta}{g} + 0$$

$$\text{(or) } R = \frac{u^2\, 2\sin\theta\cos\theta}{g}$$

$$\text{(or) } R = \frac{u^2\sin 2\theta}{g}$$ —(2) which is the required expression.

Condition for maximum horizontal range.

We have ~~$R = u^2 \sin\theta$~~ $R = \frac{u^2 \sin 2\theta}{g}$

The value of R depends on $\sin 2\theta$ keep u constant.
R will be maximum when $\sin 2\theta$ is maximum.
Maximum value of $\sin 2\theta$ is 1.

$\sin 2\theta = 1$

$\sin 2\theta = \sin 90°$

$2\theta = 90°$

$\theta = 90/2$

$\theta = 45°$

This shows when a projectile is fired at an angle 45° it covers maximum horizontal range.

$\therefore$ Maximum horizontal range is given by $R_{max} = \frac{u^2}{g}$

Problem - 1

Calculate the horizontal range covered by a stone thrown with velocity of 4.9 m/s, making an angle 15° with horizontal.

Soln

~~u~~ = Given :- $u = 4.9\ m/s$.

$\theta = 15°$

$g = 9.8\ m/s^2$

$R = ?$

We have, $R = \frac{u^2 \sin 2\theta}{g}$

$= \frac{(4.9)^2 \sin(2 \times 15)}{9.8}$

$$= \frac{\cancel{4.9}^{1} \times 4.9 \times \sin 30^\circ}{\cancel{9.8}_{2}}$$

$$= \frac{4.9 \times 1/2}{2/1}$$

$$= 4.9 \times \frac{1}{2} \times \frac{1}{2}$$

$$= \frac{4.9}{4}$$

$$= 1.225\,m.$$

Problem - 2

Find the maximum horizontal range of a projectile project with velocity $10\,m/s^{-1}$. ($g = 10\,m/s^2$)

Soln

Given :- $u = 10\,ms^{-1}$

$g = 10\,m/s^2$

$R_{max} = ?$

We have $R_{max} = \frac{u^2}{g}$

$$= \frac{(10)^2}{10}$$

$$= \frac{10\cancel{0}}{1\cancel{0}}$$

$$= 10\,m.$$

Problem - 3

A projectile is projected at an angle 45° with a velocity of 10 m/s. Find

i) Time flight

ii) Maximum height

iii) Horizontal Range.

Soln

Given :- $\theta = 45°$

$u = 10 \, m/s$

$g = 9.8 \, m/s^2$

i) Time of flight = ?

$$T = \frac{2u\sin\theta}{g}$$

$$= \frac{\cancel{2} \times 10 \times 1/\sqrt{2}}{\cancel{9.8} \; 4.9}$$

$= \cancel{1.73 \text{ sec}}$ 1.44 sec

ii) Maximum height = ?

$$H = \frac{u^2 \sin^2\theta}{2g}$$

$$= \frac{10 \times 10 \times 1/2}{2 \times 9.8}$$

$= 2.55 \, m$

iii) Horizontal Range = ?

$$R = \frac{u^2 \sin 2\theta}{g}$$

$$= \frac{(10)^2 \times \sin(2 \times 45^\circ)}{9.8}$$

$$= \frac{100 \times \sin 90^\circ}{9.8}$$

$$= \frac{100 \times 1}{9.8}$$

$$= 10.204\, m.$$

Gravitation

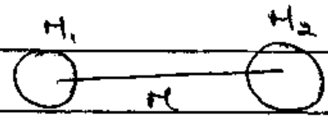
Gravitation is a natural phenomenon by which virtue of which all things which have mass attracted one another including planets, stars, galaxies, and even light and sub atomic partic

Newton's law of gravation / Universal law of gravitation.

Statement :- This law state that the attractive force between any two bodies is directly porportional to the product of theire masses and inversely proportional to the square of distance between them.

Explanation

Considere two bodies of masses M_1 and M_2 separated by a distance 'r'.



Let F be the force of attraction between them and is called gravitational force.

By this law, $F \propto M_1 M_2$ — (1)

& $F \propto 1/r$ — (2)

Combining equation – (1) and equation – (2)

$$F \propto \frac{M_1 M_2}{r^2}$$

(or) $F = G\, M_1 M_2 / r_2$ — (3)

Where G is a proportionality constant and is called universal Gravitational constant.

Universal Gravitational Constant (G)

Definition

We have, $F = G \dfrac{M_1 M_2}{r^2}$

$$\Rightarrow G M_1 M_2 = F r^2$$

$$\Rightarrow G = \frac{F r^2}{M_1 M_2}$$

where, $M_1 = M_2 = 1$ unit and $r = 1$ unit

$G = F$

Gravitational constant (G) may be defined as the force of attraction between two bodies ~~of~~ each of unit mass and separated by a unit distance.

→ S.I unit of G is $\dfrac{Nm^2}{kg^2}$ (or) $Nm^2 kg^{-2}$

→ Its dimensional formula is $[G] = \dfrac{[F] \times [r^2]}{M_1 \times M_2}$

(or) $[G] = \dfrac{[M L T^{-2}] \times [L^2]}{[M] \times [M]} = \dfrac{[M^1 L^3 T^{-2}]}{[M^2]}$

$$\Rightarrow [G] = [M^{-1} L^3 T^{-2}]$$

Acceleration due to gravity (g)

Acceleration results in a body due to Earth's gravitational force is called as acceleration due to gravity.

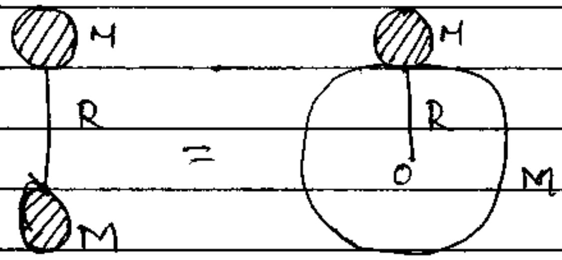
→ It is denoted by 'g'.

→ Its S.I unit is m/s^2.

→ Its dimensional formula is $[g] = [L T^{-2}]$

Relation between g & G

Consider a body of mass m is placed on the surface of the mass M and radius R.



The gravitational force between m & M is $F = G\frac{mM}{R^2}$ — (1)

By definition of force,

F = mass × acceleration

(or) $F = mg$ — (2)

From equation -(1) and equation -(2), we get

$$\cancel{m}g = G\frac{\cancel{m}M}{R^2}$$

$$(\text{or})\ g = \frac{GM}{R^2} \quad — (3)$$

Which is the required relation.

$M \rightarrow$ Mass of the earth.

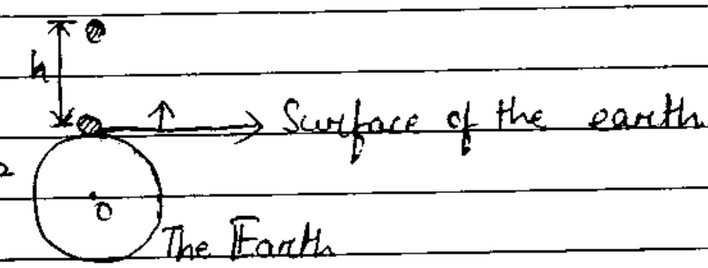
$R \rightarrow$ Radius of the earth.

Variation of 'g' with height

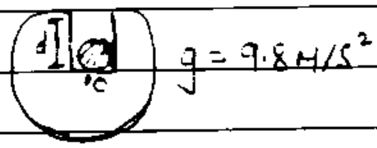
The acceleration of gravity decreases with height from the surface of the Earth. The acceleration due to gravity $\propto \frac{1}{h^2}$

(or) $g' \propto \frac{1}{h^2}$ — (1)

h - height from the surface of the earth.



Variation of 'g' with depth

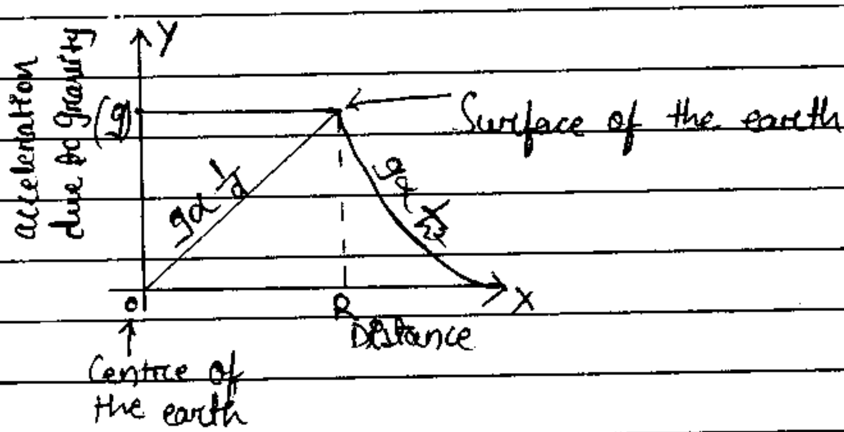


The acceleration due to gravity decreases with depth from the surface of the earth and becomes zero at the centre of the earth.

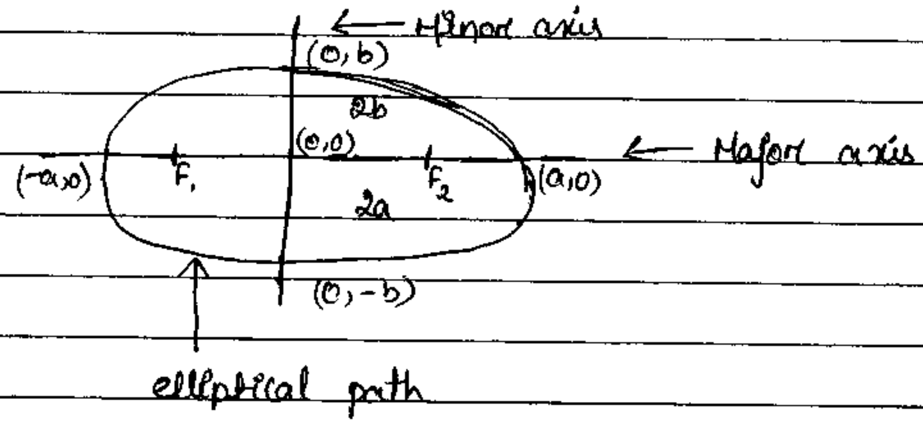
$\therefore g' \propto \frac{1}{d}$ —— (2)

Where, $d \rightarrow$ depth from the surface of the earth.
When $d = R$, the $g' = 0$.

Variation of acceleration due to gravity with height and depth in a graph.



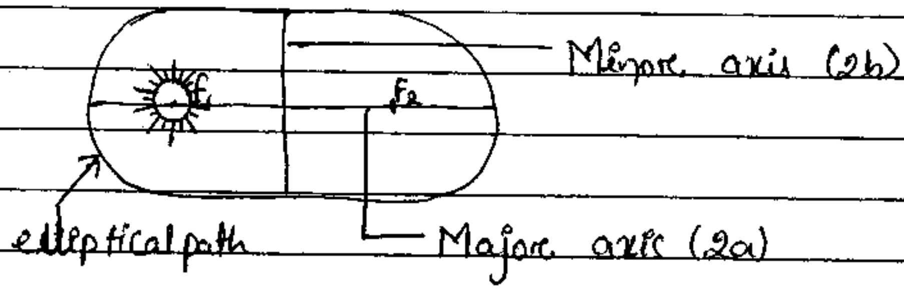
Kepler's laws of planetory motion



The German Astronomer J. Kepler have been proposed three lau
that governs the planetory motion.

1^{st} law

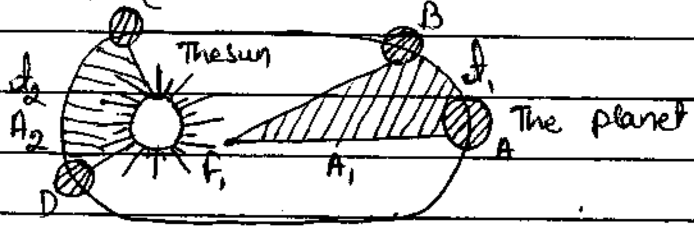
Statement :- The path of each planet about the sun is an ellipse with the sun is situated at one of the foci.



F_1 and F_2 are the foci of the ellipse.

2^{nd} law

Statement :- The distance between the sun and the planet swee
equal area in equal time interval.



In the given figure let A_1 be the area covered by the distance between the sun and the planet in time t_1. and A_2 be the area covered by the distance between the sun and the planet in time t_2.

If $t_1 = t_2$, then $A_1 = A_2$

3rd law
Statement :- The square of the given time period of a planet revolving around the sun is proportional to the cube of the length of the semi measured axis.

i.e., $T^2 \propto a^3$
where, $T \rightarrow$ Time period
$a \rightarrow$ length of semi measured axis.

Mass
Mass of many object refers to the matter contained in that object

→ It is denoted by M
→ Its S.I unit is kilogram (Kg)
→ Its DF is [M]
→ It is a scalar quantity.

Weight
Weight of any object is the force experienced by that object due to the gravity of the earth.

→ It is denoted by W.
and $w = Mg$
$M \rightarrow$ Mass of the object
$g \rightarrow$ acceleration due to gravity
→ Its S.I unit is Newton (N).

Note:-

When weight is expressed in Kg, it is actually means Kg Force (KgF). and 1KgF = 9.8 N

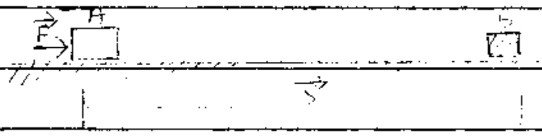
→ Its DF is $[MLT^{-2}]$.

→ It is a vector quantity and is always towards the earth.

Work And Friction

Work

Defination :- Work may be defined as the dot product of the force on a body and the displacement of the body due to the force



Let $\vec{F}$ be the force on a body, $\vec{S}$ be the displacement of the body due to the force $\vec{F}$.

$\therefore$ Work = Dot product of $\vec{F}$ & $\vec{S}$.

(or) $W = \vec{F} \cdot \vec{S}$

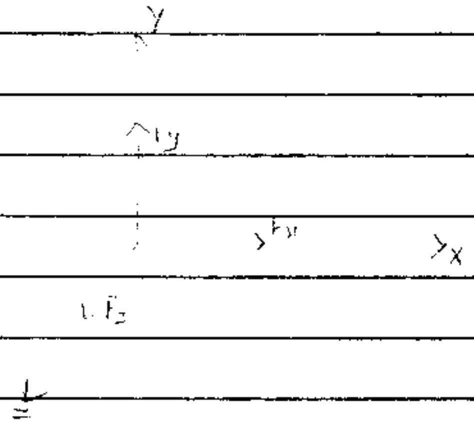
(or) $W = FS\cos\theta$

θ is the angle between $\vec{F}$ and $\vec{S}$.

(or) $W = F_x S_x + F_y S_y + F_z S_z$

F_x, F_y & F_z → Rectangulare Components of $\vec{F}$.

S_x, S_y & S_z → Rectangulare Components of $\vec{S}$.



→ Work is a scalare quantity

→ It is denoted by W.

→ Its S.I unit is Newton x meter ~~(Nm~~ (Nm)

→ Its DF is $[W] = [F] \times [S]$

$= [MLT^{-2}] \times [L]$

$[W] = [ML^2T^{-2}]$

Problem - 1

Find the work done by a force of 10 N when displacement by the force is 5m and the angle between force and displacement is 90°.

Soln

Given :- $F = 10N$

$S = 5m$

$\theta = 90°$

We have $W = FS\cos\theta$

$= 10 \times 5 \times \cos 90°$

$= 50 \times 0$

$= 0$ Nm.

Friction

Introduction

When two surfaces are in contact with each other, there exist a force between these surface and is called contact force.

This contact force opposes the motion of a surface over the other and is called frictional force (or) friction.

Defination

Whenever a body slides (or) attempt to slide over a surf the motion is registed by the bonding between the surf and the opposing force which resist this motion is called frictional force (or) friction.

Types Of friction

There are two types of friction as follow

Friction

- Static friction
- Dynamic friction (or) Kinematic friction

Static friction

Defination

Static friction is the force of friction between two surfaces so long as there is no relative motion between them.

(Or)

Static friction is the force of friction between two surfaces when both surfaces are at rest.

→ It is denoted by f_s.

→ Static friction is a self - adjustable.

→ The maximum static friction between the surface is called limiting friction.

Limiting Friction

Defination

Limiting friction may be defined as the maxinum value of static friction between two surfaces.

→ It is denoted by $f_s max$

Dynamic / Kenematic friction.

Defination

Dynamic friction is a force of friction between two surfaces when one (or) ~~more~~ both the surface are in motion.

Laws of friction (or) laws of limiting friction.

i) The direction of friction (or) limiting friction is always opposite to the direction of motion.

ii) The friction (or) limiting friction is proportional to the normal force / normal reaction between the two bodies.

Mathematically,

$F \propto R$

(or) $F_{s,max} = \mu_s R$

Where μ_s is a proportionally constant called co-efficient of static friction.

iii) The friction (or) limiting friction depends upon the state of polish of the surfaces.

Co-efficient of friction

We have,

$F \propto R$

(or) $F = \mu R$

Where, μ is a proportionality constant and called co-efficient of friction.

(or) $\mu = \frac{F}{R}$

Definition.

Co-efficient of friction between a pair of surfaces is defined as the ratio force of friction and normal reaction.

→ It is unitless as well as dimensionless.

Ex-1

Find the co-efficient of friction between two surfaces if the force of friction is 10N and the normal reaction is 2N.

Given :- Force = 10N

R = 2N

We have $\mu = \frac{F}{R}$

$= \frac{\cancel{10}^{5} \cancel{N}}{\cancel{2} \cancel{N}}$

$= 5$

Friction is a necessary evil. Justify

Advantages of friction.

→ We are able to hold the pens during writing due to the friction between the pens and the fingers.

→ We are able to stop automobiles, bikes, cycles etc due to friction.

→ We are able to walk on a floor due to friction

Disadvantages of friction.

→ There is wastage of energy due to friction.

→ Wear & tear (damage) of different parts of a machine due to friction.

→ There are wastage of materials due to friction.

Since, friction has both advantages as well as disadvantages it is called a necessary evil.

Meathods to Reduce the Friction.

→ By using lubricants (e.g., oil, greec etc)
→ By polishing the surfaces.
→ By converting sliding friction into rolling friction.
→ By streamlining

Oscillations and waves

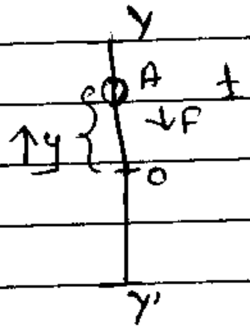
Simple Harmonic Motion (SHM)

→ It is a to and fro motion about a fixed point called mean position in SHM

Definition

Simple harmonic motion is a type of motion in which the restoring force is proportional to the displacement and directed opposite the displacement.

Suppose a body is performing a SHM along a vertical line YY' about the mean position 'O'.



Then by definition of SHM,

Restoring force $\propto$ - displacement

$F \propto -y$

or, $F = -ky$

Where, k is a proportionality constant.

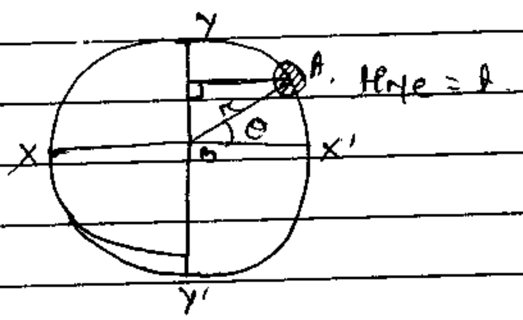
Examples of SHM

→ Motion of a simple pendulum

→ Vibration of a loaded spring.

→ Bouncing of a ball.

Relation between SHM & Circular Motion

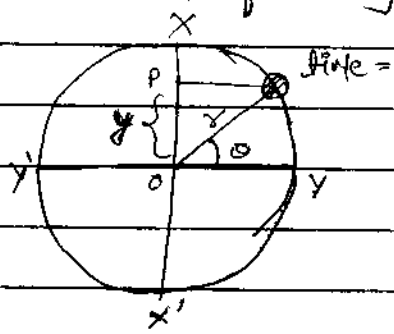


Consider a body is undergoing in a circular motion along a circular path of radius 'r'.

XOX' → Horizontal diameter
YOY' → ~~the~~ Vertical diameter
let the body is at ~~a~~ 'A' at any ~~point~~ instant 't'.

Draw a perpendicular from the point A on the diameter YOY'
let 'P' be the foot of perpendicular and is called projection
of the body.
It is noted that when the body is undergoing circular
motion, its projection performs SHM about the centre
'O' along the vertical diameter YOY'.

Expressions for displacement, velocity and acceleration of a body
in SHM.



Displacement :- Displacement of a body
vibrating in SHM at any ~~point~~ instant
is defined as ~~the~~ its distance from mean
position at that instant.

Let y be the displacement of projection 'P' of the body at
time 't', as shown in the given figure.

In the right angled $\triangle OPA$

$\sin\theta = \dfrac{OP}{OA}$

or, $\sin\theta = \dfrac{y}{r}$

or, $y = r\sin\theta$

we have $\omega = \dfrac{\theta}{t}$

or, $\theta = \omega t$

Substituting $\theta = \omega t$ in equation - 1

$\therefore y = r \sin \omega t$ — (2)

which is the required expression for displacement of a body in SHM.

where, $r \rightarrow$ amplitude of SHM.
$\omega \rightarrow$ Angular velocity

Velocity :- Velocity of a body in SHM is the rate of change of its displacement with time.

i.e., $v = \dfrac{dy}{dt}$

or, $v = \dfrac{d}{dt}(r \sin \omega t)$

or, $v = r \dfrac{d}{dt}(\sin \omega t)$

or, $v = r \cos \omega t \times \dfrac{d}{dt}(\omega t)$

or, $v = r \cos \omega t \times \omega \times \cancel{\dfrac{dt}{dt}}\,1$

or, $v = r\omega \cos \omega t$ — (3)

or, $v = r\omega \cos \theta$ — (4)

$y = \sin t$

$\dfrac{dy}{dt} = \dfrac{d}{dt}(\sin t)$

$= \cos t \times \dfrac{d}{dt}(t)$

$= \cos t$

3

In the right angled ΔOPA,

$$\cos\theta = \frac{AP}{OA} \quad -(5)$$

~~or, $\cos\theta =$~~

and $OA^2 = OP^2 + AP^2$

$$\Rightarrow AP = \sqrt{r^2 - y^2} \quad -(6)$$

$$\therefore \cos\theta = \frac{\sqrt{r^2 - y^2}}{r} \quad -(7)$$

Substituting equation $-(7)$ in equation $-(4)$

$$\therefore V = \cancel{r}\omega \times \frac{\sqrt{r^2 - y^2}}{\cancel{r}}$$

$$\text{or, } V = \omega\sqrt{r^2 - y^2} \quad -(8)$$

which is the required expression for velocity of a body in SHM.

At extreme positions.

$y = \pm r$

we have

$$V = \omega\sqrt{r^2 - y^2}$$

$$\text{or, } V = \omega\sqrt{r^2 - r^2}$$

$$\text{or, } V = \omega \times 0$$

$$\text{or, } V = 0$$

$\therefore$ This shows the velocity of a body is ~~zero~~ zero at extreme posi

At mean position,

$y = 0$

$\therefore v = \omega\sqrt{r^2 - 0^2}$

$v = \omega\sqrt{r^2}$

or, $v = \omega r$ (Maximum)

This shows the velocity of a body is maximum at mean position.

Acceleration :- Acceleration of a body in SHM is the rate of change of velocity.

i.e., $a = \frac{dv}{dt}$

or, $a = \frac{d}{dt}(r\omega\cos\omega t)$ [from equation -(3)]

or, $a = r\omega\frac{d}{dt}(\cos\omega t)$

or, $a = r\omega \times -\sin\omega t \times \frac{d}{dt}(\omega t)$

or, $a = -r\omega\sin\omega t \times \omega \times \frac{\cancel{dt}}{\cancel{dt}}1$

or, $a = -r\omega^2\sin\omega t$ — (9)

or, $a = -r\omega^2\sin\theta$ — (10)

In the right angled ΔOPA

$$\sin\theta = \frac{OP}{OA}$$

or, $\sin\theta = \dfrac{y}{r}$

$\therefore$ equation $-(10)$ becomes.

$$\therefore a = -r\omega^2 \times \frac{y}{r}$$

$$a = -\omega^2 y \quad -(11)$$

which is the required expression for the accelaration of a body in SHM.

At mean position

$y = 0$

$\therefore \; a = -\omega^2 \times 0 = 0$

This shows acceleration of a body in SHM is zero at mean position.

At extreme position

$y = \pm r$

$\therefore a = -\omega^2 \times (\pm r)$

Or, $a = \pm \omega^2 r$

Wave Motion

Defination:-

Wave motion may be defined as the disturbance travel in a medium due to repeated periodic motion of the particles of th medium and the disturbance is handed over from one particle to other.

On the basis of the motion of the particles of a medium the wave motion can be classified into two types as follow

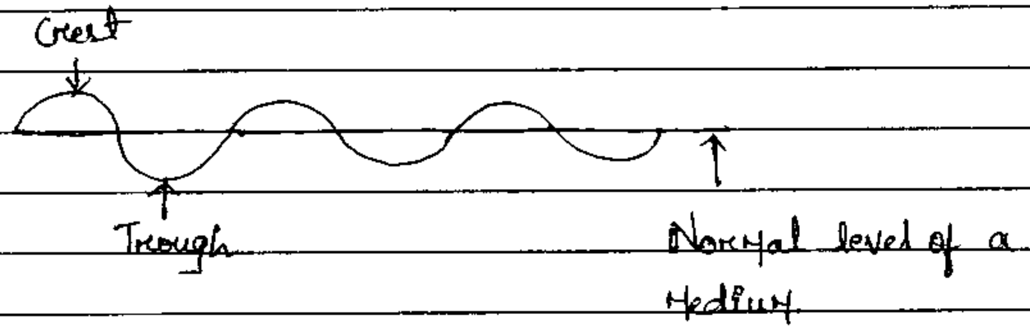
Wave

Transverse wave | [illegible]

Transverse wave :- Transverse wave is a type of wave in which particles of a medium are vibrating in a direction at right angle to the propagation of the wave.
Example :- Water wave.

Crest :- When a transverse wave travels ~~to~~ through a medium some portions of the medium get raised from the normal level of the medium and are called crest.

Trough :- When a transverse wave travels through a medium some portions of the medium get depressed from the normal level of the medium and are called trough



(Propagation of a transverse wave)

Longitudinal wave :- Longitudinal wave is a type of wave in which particles of a medium are vibrating in the same direction as that of the propagation of the wave.
Example :- Sound wave / Sound

Compression :- When a longitudinal wave travells in a medium, some protions of the medium get compressed and are called compression.

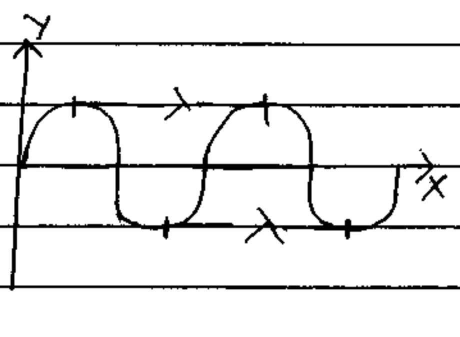
Rarefaction :- When a longitudinal wave travels in a medium, some portions of the medium get rarefied (less-densed) and are called rarefaction.

C R C R C

Some terms connected to wave motion.

1) Time period :- It is the time taken by a particle of a medium to perform a complete vibration.
→ It is denoted by T.

ii) Wave length :- It is the distance travelled by a wave in the time T (Time period) is called wave length.



→ It is denoted by λ (lambda).

iii) Wave velocity :- It is the distance travelled by a wave divided by the time interval
→ It is denoted by 'v'.

iv) Frequency :- It is the reciprocal of time period.
→ It is denoted by f (or) ν (neu)

$$\therefore f \text{ (or) } \nu = \frac{1}{T}$$

Relation between velocity, frequency and wave length of a wave.

We have, wave length of a wave is the distance travelled by the wave in time T (time period of the wave).

Let λ be the wave length of the wave. By defination, the wave velocity is given by

$$v = \frac{\text{Distance}}{\text{Time}}$$

$$\text{(or) } v = \frac{\lambda}{T}$$

$$\text{(or) } v = \lambda\left(\frac{1}{T}\right)$$

$$\text{(or) } v = \lambda f$$

which is the required expression.

Ultrasonic

Human ear is responsible to only a particulare frequency range lying in between 20 Hz to 20,000 Hz and is called audible range.

→ Sound of frequency greater than upper limit of audible range (i.e., 20,000 Hz) is called ultrasonic.

Properties of Ultrasonic.

→ It is longitudinal in nature.
→ Propagation of a ultrasonic in a medium produces compressions and rarefaction in that medium.
→ It has a frequency range of ~~2x~~ 2×10^4 Hz to 10^9 Hz.
→ It is a high energetic wave.

Uses of Ultrasonic

→ Ultrasonic welding
→ Ultrasonic cleaning
→ Medical uses (Diagonstic uses)
→ Study of microstructure.
→ Ultrasonic ~~therapy~~ theraphy
→ Determination of elastic constant.

Heat and thermodynamics

Heat & temperature

Heat:-

Heat is the mode of transfer of energy

→ S.I unit of heat is Joule (J).

→ Another unit of heat is Calorie.

→ 1 calorie = 4.18.

Temperature

Temperature is the measure of average kinetic energy o particles of a matter.

→ S.I unit of temperature is kelvin (k).

Thermal Expansion

Thermal expansion is the tendency of matter (solid, li gas & plasma) to change in size, area and volume in respond to change in temperature

Thermal expansion in solids.

Unlike other states of matter, solids also expand on heating.

~~Co-efficient of thermal expansion~~

When a body is heated, it expands in all dimensions i.e along its length, breadth and thickness simultaneously.

Thermal Expansion

- Linear expansion
- Superficial expansion
- Cubical expansion

Linear expansion

It is the thermal expansion of a solid in one dimension and is also called as one dimensional expansion

Consider a long thin rod which may be a one dimensional object.

Let l_0 is the length of the rod at $0^\circ c$.
On heating the rod will expand and let l_t be the length of the rod at $t^\circ C$.
The change in length of the rod is $l_t - l_0$.
The change $l_t - l_0$ depends on following two factors.

i) $l_t - l_0 \propto l_0$ — (1)
ii) $l_t - l_0 \propto t$ — (2)

Combining eq^n -(1) and eq^n (2), we get
$l_t - l_0 \propto l_0 t$
or, $l_t - l_0 = \alpha l_0 t$ — (3)

where, α is a proportionality constant and is called co-efficient of linear expansion.
or, $l_t = l_0 + \alpha l_0 t$
or, $l_t = l_0 (1 + \alpha t)$ — (4)

From eq^n-(3)
$$\alpha = \frac{l_t - l_0}{l_0 t} \quad \text{— (5)}$$

when $l_0 = 1$ and $t = 1^\circ c$
$\alpha = l_t - l_0$

Definition:-
Co-efficient of linear expansion may be defined as the change in length per unit length at 0°C per 1°c rise in temperature.

Superficial expansion.
It is the thermal expansion of a solid in two dimensions and is also called as two dimensional expansional

Consider a thin sheet of metal which can be consider as a two dimensional object.

Let S_0 be the area of the sheet at 0°c.
On heating it expands along two dimensions, let S_t be the area of the sheet at t°c.

∴ The change in area is $S_t - S_0$ and it depends on two factors.

i) $S_t - S_0 \propto S_0$ —— (1)

ii) $S_t - S_0 \propto t$ —— (2)

Combining eq^n (1) and eq^n (2), we get

$S_t - S_0 \propto S_0 t$

or, $S_t - S_0 = \beta S_0 t$ —— (3)

Where, β is proportionality constant and is called co-efficient of superficial expansion.

or, $S_t = S_0 + \beta S_0 t$

or, $S_t = S_0 (1 + \beta t)$ —— (4)

Again from eq^n – (3)

$$\beta = \frac{S_t - S_0}{S_0 t} \quad \text{——} (5)$$

When $S_0 = 1$, $t = 1^\circ c$, then $\beta = S_t - S_0$

Def^n

Co-efficient of superficial expansion of a solid may be defined as change in area per unit area at $0^\circ c$, per $1^\circ c$ rise of temperature.

Cubical expansion.

It is the thermal expansion of a solid in all three dimensions and is the also called the three dimensional expansion.

Consider a ~~cub~~ iron cube.

let V_0 be the volume of the cube at $0^\circ c$.

On heating, it expands in all dimensions let V_t be the volume at $t^\circ c$.

∴ The change in volume is $V_t - V_0$ and it depends on following two factors

i) $V_t - V_0 \propto V_0$ – (1)

ii) $V_t - V_0 \propto t$ – (2)

Combining eq^n (1) and eq^n (2), we get

$V_t - V_0 \propto V_0 t$

or, $V_t V_0 = \gamma V_0 t$

where γ is a proportionality constant and is called Co-efficient of cubical expansion.

or, $V_t = V_0 + \gamma V_0 t$

or, $V_t = V_0 (1 + \gamma t)$ —— (4)

Again from eqⁿ — (3),

$$\gamma = \frac{V_t - V_0}{V_0 t}$$ —— 5

When $V_0 = 1$ and $t = 1°c$ then $\gamma = V_t - V_0$

Defⁿ

Co-efficient of cubical expansion of a solid may be defined as the change in volume per unit volume at 0°c, per 1°c rise in temperature.

* Relation between α and β

~~Considere~~ Consider a square sheet have length l_0 at 0°c

∴ Area S_0 at 0°c is

$S_0 = l_0^2$ —— (1)

On heating length of the square changes.

Let l_t be the length of the square at t°c.

∴ Area ~~St~~ S_t at t°c is $S_t = l_t^2$

or, $S_t = [l_0 (1 + \alpha t)]^2$ ($\because l_t = l_0 (1 + \alpha t)$)

or, $S_t = l_0^2 (1 + \alpha t)^2$ — (2)

We have

$$\beta = \frac{S_t - S_0}{S_0 t}$$

Or, $\beta = \frac{l_0^2 (1+\alpha t)^2 - l_0^2}{l_0^2 t}$

Or, $\beta = \frac{l_0^2 (1+\alpha t)^2 - 1}{l_0^2 t}$.

Or, $\beta = \frac{1+\alpha t^2 - 1}{t}$

Or, $\beta = \frac{1^2 + 2 \times 1 \times \alpha t + (\alpha t)^2 - 1}{t}$

Or, $\beta = \frac{2\alpha t + \alpha^2 t^2}{t}$

Or, $\beta = \frac{t(2\alpha + \alpha^2 t)}{t}$

Or, $\beta = 2\alpha + \alpha^2 t$

Since α is very small, $\alpha^2 t$ can be neglected in the abo
equation

$\beta = 2\alpha$

$\Rightarrow \alpha = \beta/2$ — (3)

Relation between α and γ

Considere a cube having each side l_0 at 0°C.

$\therefore$ Volume V_0 at 0°c, $V_0 = l_0^3$ —— (1)

On heating, each side of the cube expands,

let l_t be the length of each side of the cube at t°c.

$\therefore$ Volume V_t at t°c, $V_t = l_t^3$ —— (2)

Or, $V_t = [l_0 (1+\alpha t)]^3$ $\quad [\because l_t = l_0 (1+\alpha t)]$

or, $V_t = l_0^3 (1+\alpha t)^3$

~~or, $V_t = l_0^3 \; 1^3 + 3\times1\times\alpha t +$~~

We have,

$$\gamma = \frac{V_t - V_0}{V_0 t}$$

or, $\gamma = \dfrac{l_0^3 (1+\alpha t)^3 - l_0^3}{l_0^3 t}$

or, $\gamma = \dfrac{\cancel{l_0^3}\left[(1+\alpha t)^3 - 1\right]}{\cancel{l_0^3}\, t}$

or, $\gamma = \dfrac{(1+\alpha t)^3 - 1}{t}$

or, $\gamma = \dfrac{1^3 + 3\times 1^2 \times(\alpha t) + 3\times 1\times(\alpha t)^2 + (\alpha t)^3 - 1}{t}$

or, $\gamma = \dfrac{\cancel{1} + 3\alpha t + 3\alpha^2 t^2 + \alpha^3 t^3 \cancel{-1}}{t}$

or, $\gamma = \dfrac{\cancel{t}\,(3\alpha + 3\alpha^2 t + \alpha^3 t^2)}{\cancel{t}}$

or, $\gamma = 3\alpha + 3\alpha^2 t + \alpha^3 t^2$

Since, α is very small $3\alpha^2 t$ and $\alpha^3 t^2$ can be neglected in the above equation, we get

$\gamma = 3\alpha$

or $\alpha = \gamma / 3$ —— (3)

We have $\alpha = \frac{\beta}{2}$

$\alpha = \frac{\gamma}{3}$

$\therefore \alpha = \frac{\beta}{2} = \frac{\gamma}{3}$

Which is the required relation between α, β and γ.

Problem - 1

A piece of copper wire have a length of 10 m at 0°c. Find its length at 80°c. Given $\alpha = 17 \times 10^{-6}\,{}^\circ c^{-1}$

Given :-

length(l_0) = 10 m

$t = 80°c$

$l_t = l_{80} = ?$

We have,

$l_t = l_0 (1 + \alpha t)$

or, $l_{80} = l_0 (1 + \alpha t)$

$= 10 (1 + 17 \times 10^{-6} \times 80)$

$= 10 (1 + 1360 \times 10^{-6})$

$= 10 \left[1 + \frac{1360}{1000000}\right]$

$= 10 [1 + 0.00136]$

$= 10 [1.00136]$

$= 10.0136$ m

Problem - 2

The volume of a lead ball is $10^{-5} m^3$ at $0°c$ and $1.005 \times$ at $100°c$. Calculate the co-efficient of linear expansion.

Given :- $V_0 = 10^{-5}$

$V_{100} = 1.005 \times 10^{-5} m^3$

$\alpha = ?$

We have, $\gamma = \frac{V_t - V_0}{V_0 t}$

or, $\gamma = \frac{V_{100} - V_0}{V_0 \times 100}$

$\gamma = \frac{1.005 \times 10^{-5} - 10^{-5}}{10^{-5} \times 100}$

or, $\gamma = \frac{10^{-5}(1.005 - 1)}{10^{-3}}$

or, $\gamma = 10^{-5} \times 0.005 \times 10^{3}$

or, $\gamma = 0.005 \times 10^{-2}\ °c^{-1}$

$\alpha = \frac{\gamma}{3}$

or, $\alpha = \frac{0.005 \times 10^{-2}}{3}$

or, $\alpha = 0.0016 \times 10^{-2}\ °c^{-1}$

10-5-1

Mechanical equivalent of heat

Mechanical equivalent of heat deals with work equivalence of heat.

According to this, if W be the work be the work done to produce Q amount of heat.

then,

$W \propto Q$

or, $W = JQ$

Where J is a proportionality constant and is called Joule's mechanical equivalent of heat.

or, $J = \frac{W}{Q}$

When $Q = 1$, $J = W$

Defination of Joule's mechanical equivalent of heat ~~may~~

Joule's mechanical equivalent of heat may be defined as the amount of work which produces unit quantity of heat.

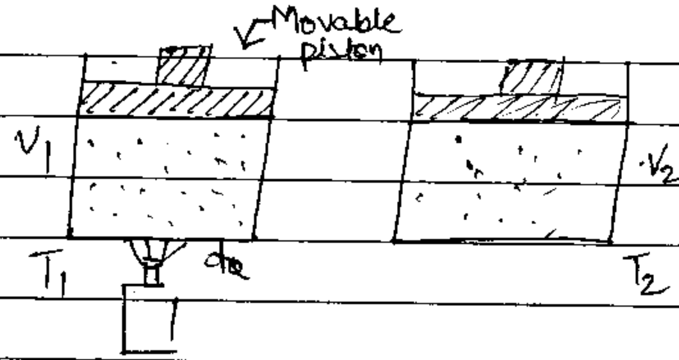
→ J is a dimensionalless quantity.

State and explain 1st law of thermodynamics

1st law of thermodynamics.

Statement :- 1st law of thermodynamics states that if dQ amount of heat is given to a system and dw be the amount of work done by the system, then the change in internal energy of the system is given by $du = dQ - dw$

or, $dQ = du + dw$



Specific heat

If Q is the amount of heat given to a body of mass 'm' and ΔT be the change in temperature, then $Q \propto m$ — (1)

$Q \propto \Delta T$ — (2)

Combining eqⁿ — (1) and eqⁿ — (2), we get

$$Q \propto m \Delta T$$

$$Q = C m \Delta T \quad — (3)$$

Where C is a proportionality constant and is called specific heat of the material of the body

or, $$C = \frac{Q}{m \Delta T} \quad — (4)$$

When, $m = 1$, $\Delta T = 1\,k$, $C = Q$

Defⁿ

Specific heat of the material of a body may be defined as the amount of heat required to raise the temperature of unit mass of the material by 1 k.

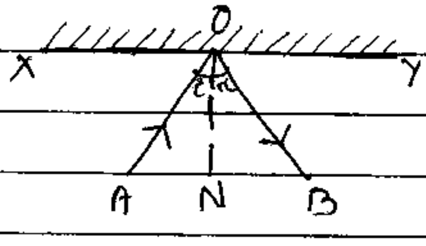
Dimensional formula of 'C' is $L^2 T^{-2} k^{-1}$.

Optics

Reflection :-

Defⁿ

Reflection is the phenomenon by virtue of which a ray of light is sent back to the same medium from which it is coming after being obstracted by a surface.



AO → Incident ray

OB → Reflected ray

XY → Reflecting Surface.

ON → The normal to the reflecting surface at the point of reflection 'O'.

O → ~~Reflection~~ point of reflection

Angle of incidence ($\hat{i}$) → It is the angle between incident ray and the normal to the reflecting surface at the point of reflection.

~~r → It is~~

r → Angle of reflection :- It is the angle between reflected ray and the normal to the reflecting surface at the point of reflection.

Laws of reflection

There are two laws of reflection

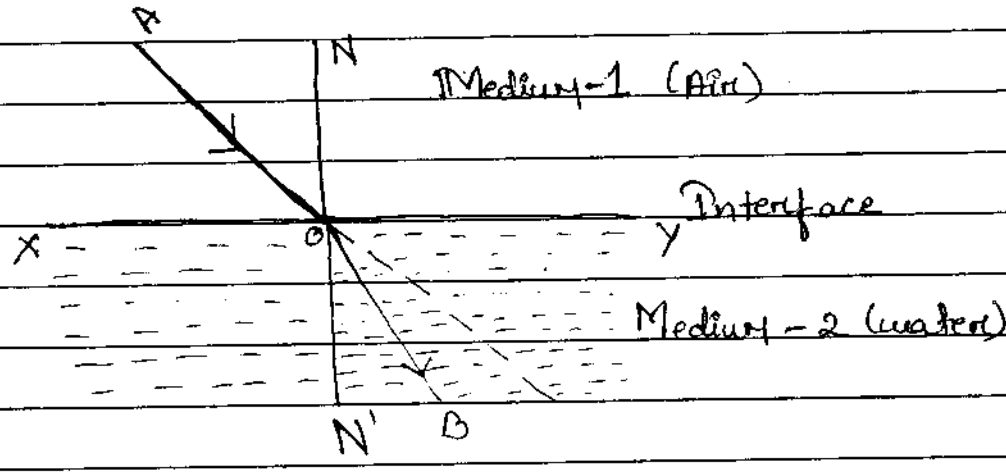
i) The incident ray, the reflected and the normal to the reflecting, all ~~line~~ lie in one plane and that plane is perpendiculare to the reflecting surface.

ii) Angle of incidence is equal to angle of reflection.

i.e., $\angle i = \angle r$

Refraction

Def^n :- Refraction is a phenomenon in which a ray of going from one medium to other undergoes a change velocity.



AO → Incident ray

XY → Interface :- The surface / line separating two me

NON' → The perpendicular to the interface XY at the of interface 'O'.

OB → Refracted ray.

i :- Angle of incidence

~~OB~~ r :- Angle of Refraction

Laws of Refraction

There are two laws of refraction :

i) The sine of the angle of incidence bears a constant ratio with sine of the angle of refraction

i.e. $\frac{\sin i}{\sin r}$ = constant

The law is often termed as Snell's law.

ii) The incident ray, the refracted ray and the normal to the interface at the point of incidence all lie in one plane and that plane perpendicular to the interface separating the two medium

Refractive index

Refractive index of a medius is the medium of speed of light in that medium

Def^n

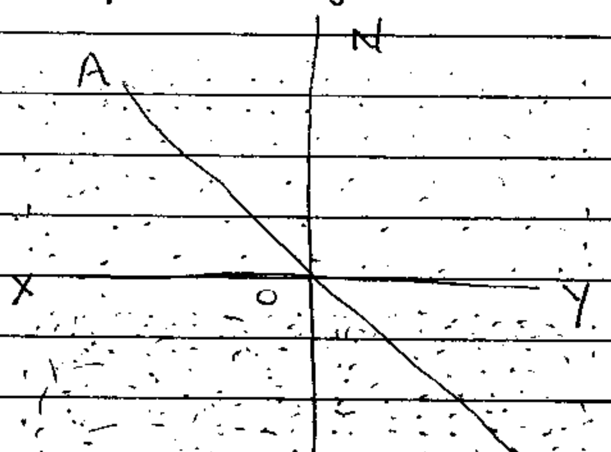
Refractive index of a medium is the Ratio index of a speed of the light in vaccume to the speed of the light in that medium.

i.e. $\mu = \frac{c}{v}$ - $\mu \rightarrow$ MU

$\mu \rightarrow$ Refractive index of the given medium

$c \rightarrow$ Speed of the light in volume $= 3 \times 10^8 m/s$

$v \rightarrow$ Speed of the light in the given medium



Consider two medium of refractive index μ_1 & μ_2

By defination, $\mu_1 = \frac{c}{v_1}$ —— (1)

and $\mu_2 = \frac{c}{v_2}$ —— (2)

dividing eq ① by eq ②

$\frac{\mu_1}{\mu_2} = \frac{c/v_1}{c/v_2}$

or, $\frac{\mu_1}{\mu_2} = \frac{c/v_1}{c/v_2}$

or, $\frac{\mu_1}{\mu_2} = \frac{\cancel{c}}{v_1} \times \frac{v_2}{\cancel{c}}$

— ③

or $\frac{\mu_1}{\mu_2} = \frac{v_2}{v_1}$

— ④

or $\mu_1 = \frac{v_2}{v_1}$

when $\mu_2 = \mu_1$ and is called refractive index of μ_2 medium 2 w.r.t medium-1

Note

$_w\mu_g = \frac{\mu_g}{\mu_w} = \frac{v_2}{v_1}$

$_w\mu_g = \frac{\mu_w}{\mu_g} = \frac{v_1}{v_2}$

P① Find the speed of light in glass if its refractive index is $\frac{3}{2}$

Soln

Given $\mu = 3/2$

we have $\mu = c/v$

$v = ?$

$\frac{3}{2} = \frac{3 \times 10^8}{v}$

$\Rightarrow 3v = 6 \times 10^8$

$\Rightarrow v = \frac{26 \times 10^8}{3} = 2 \times 10^8$ m/s.

P.(2) The refractive index of glass w.r.t air is 1.5. If speed of light in glass is 2×10^8 m/s & the speed of light in air.

Soln $v_1 = 2\times10^8$ m/s

$v_2 = ?$

We have $_a\mu_g = {}_2\mu_1 = \frac{v_1}{v_2}$, $v_2 = ?$

$$\boxed{\begin{aligned} {}_1\mu_2 &= \frac{\mu_1}{\mu_2} = \frac{v^2}{v^1} \\ {}_2\mu_1 &= \frac{\mu_2}{\mu_1} = \frac{v_1}{v_2} \end{aligned}}$$

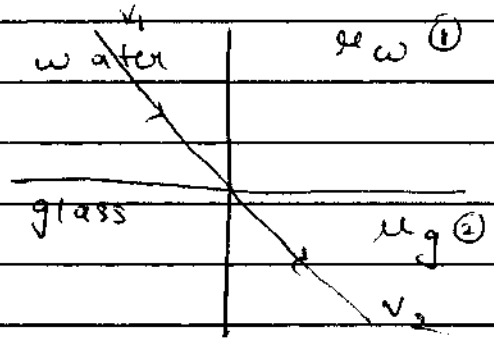
or, $_a\mu_g = \frac{v_1}{v_2}$

or $\frac{15}{10}{}^{3}_{2} = \frac{2\times10^8}{v_2}$

or, $3v_2 = 4\times10^8$

or, $v_2 = \frac{4\times10^8}{3} = 1.33\times10^8$ m/s

P.(3) The speed of light if water is 2×10^8 m/s and that of in glass is 1×10^8 m/s and. Find $_g\mu_w$ & $_w\mu_g$.



Soln given $V_1 = 2\times10^8$ m/s glass

$V_2 = 1.5\times10^8$ m/s

$_g\mu_w = \frac{v}{\mu_2} = \frac{2\times10^8}{1.5\times10^8}$

$= \frac{20}{15} = 4/3$

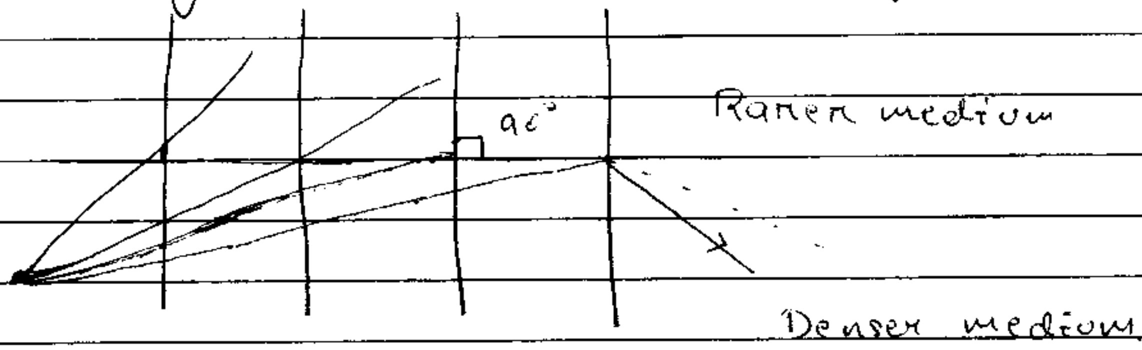
$_g\mu_w = \mu_1 = v_1/v_2$

$_w\mu_g = \mu_2 = v_2/v_1$

$_w\mu_g = \frac{V_2}{V_1} = \frac{1.5\times10^8}{2\times10^8}$

$= \frac{15}{20} = \frac{3}{4}$

Critical angle and total internal reflection



Critical angle and total internal refraction
There are two condition for total internal refration.
1) The ray must travel from denser medium to rarer medium.
2) The angle of incidence should be greater than critical angle i.e. $i > i_c$
$i_c \rightarrow$ Critical angle.

Critical angle
Critical angle is the angle of incidence in the denser medium for which the angle of refraction in the rarer medium is 90° It is denoted by i_c

Total internal reflection (TIR)
Def^n
Total internal reflection is the phenomenon by virtue of which a ray of light travelling from denser medium to rarer medium is sent back to denser medium, when the angle of incidence is greater than the critical angle of that ray.

8-2 source of light (Total internal reflection).

Illustration of total internal reflection.

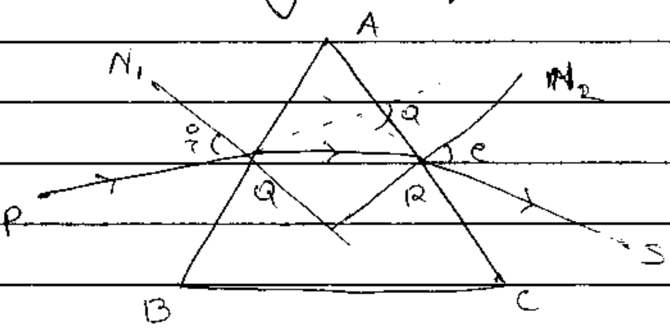
Mirage

Inferior mirage   Superior mirage

Inferior mirage.
It is an optical illusion observed in hot region.

Superior mirage
It is an optical illusion observed in cold regions

Refraction through a prism



Let ABC be the cross section of a prism. AB & BC are the two interface of the prism

PQ → Incident ray

N_1 → Normal to the interface AB at the point Q

QR → Refracted ray.

N_2 → Normal to the interface AC at the point R.

RS → Emergent ray.

i → angle of incidence.

e → angle of emergence.

d → angle of deviation. It is the angle between the incident ray & the emergent

There exist a unique minimum value d, and is called angle of minimum deviation. & is denoted by dm.

Derive an expression for refractive index the material of a prism.

* Let μ be the refractive index of the material of a prism.

$$\mu = \frac{\sin\left(\frac{A+d_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

Where $A \rightarrow$ angle of the prism / Refracting ang
$d_m \rightarrow$ angle of minimum deviation

Problem - 4

A ray of light passing through a glass prism of refracting angle 60° undergoes a minimum deviation of 30°. Calculate the refractive index of the glass prism.

$A = 60^\circ$

$d_m = 30^\circ$

$$\mu = \frac{\sin\left(\frac{60^\circ + 30^\circ}{2}\right)}{\sin\left(\frac{60^\circ}{20^\circ}\right)}$$

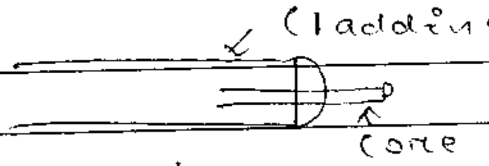
$$= \frac{\sin\left(\frac{90^\circ}{2}\right)}{\sin\left(\frac{60^\circ}{2}\right)}$$

$$= \frac{\sin(45)^\circ}{\sin(30^\circ)}$$

$$= \frac{1/\sqrt{2}}{1/2}$$

$$= \frac{1}{\sqrt{2}} \times \frac{2}{1} = \frac{1 \times 2 \times \sqrt{2}}{\sqrt{2} \times 1 \times \sqrt{2}} = \frac{2\sqrt{2}}{2} = \sqrt{2} = 1.4$$

Optical fibers



1) A optical fiber has two parts.
2) Core : It is made of glass (or) plastic.
3) cladding: It is also made up of glass (or) plastic but its refractive index is less than that of core.

Refractive index of cladding is less than refractive index of core.

ii) signal or A ray of light transmits through an optical fiber due to total internal reflection.

Uses

→ For transmission of signal like telephone signals, internet, signals at a faster rate.
→ Medical purposes.
→ Optical fibers are also used for decorative purposes.

Part-1

Unit-09: Electrostatics & Magnetostatics

Q: What is electrostatic?

Electrostatic

Electrostatic is a branch of physics that studies electric charges at rest.

Q: What is electric charge?

Electric Charge

Electric Charge is a basic property of the elementary particles of matter which is responsible for the electric force between various objects.

Q: Write types of electric charges.

Types of electric charges

There exist two types of electric charges

(i) positive charges

(ii) Negative charges

NB: By convention, the charge on an electron is considered as negative and that on a proton is considered as positive.

Q: Write SI unit and dimensional formula of electric charge.

→ SI unit of electric charge is Coulomb (C)

Other units of electric charge are as follows

(i) esu (electrostatic unit) or stat coulomb (CGS system)

$\boxed{1\text{ st.Coulomb} = 3.3\times10^{-10}\,C}$

~~(ii) emu (electromagnetic unit)~~ or ab coulomb. $\boxed{1aC = 10C}$

(iii) Ampere-hour (Ah), $i = q/t \Rightarrow \boxed{q = it}$

(iv) millicolumb (mC), $\boxed{1mC = 10^{-3}C}$

(v) Micro columb (μC), $\boxed{1\mu C = 10^{-6}C}$

(vi) Nano columb (nC) ... so on, $\boxed{1nC = 10^{-9}C}$

→ Dimensional formula of electric charge is

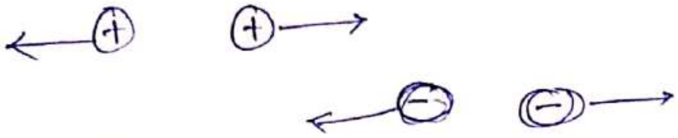
$$[q] = [Q] = [I^1T^1]$$

Q. What is electric force?

Electric force

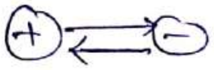
The force of attraction or repulsion between two electric charges or electric charged bodies is called electric force.

→ The nature of electri force between two like charges or like charged bodies is repulsive



(Repulsive in nature)

→ The nature of electric force between two unlike (different) charges or unliked charged bodies is attractive



(Attractive in nature)

→ Electric force is a Conservative force

Q: State and explain Coulomb's law in electrostatic

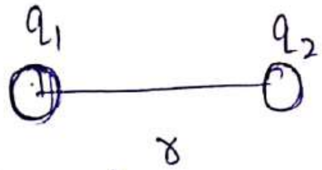
Coulomb's law

Statement: This law states that the electric force between two charged bodies is

→ directly porportional to the product of their charges

→ Inversely proportional to the square of distance between them.

Explanation



Let F is the electric force between two charged bodies of charges q_1 and q_2, placed at a distance of 'r'.

According to coulomb's law.

$$F \propto q_1 q_2 \quad \text{——} (1)$$

$$F \propto \frac{1}{r^2} \quad \text{——} (2)$$

Combining eqn – (1) & eqn (2), we get

$$F \propto \frac{q_1 q_2}{r^2}$$

$$\Rightarrow \boxed{F = K \frac{q_1 q_2}{r^2}} \quad \text{——} (3)$$

where K is a proportionality constant and the value of K depends on following two factors

(i) the nature of the medium between two charged bodies

(ii) the system of units choosen to measure F, q_1, q_2 & r.

∴ when two charged bodies are placed in free space/vacuum or air then,

$$\boxed{F = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}} \quad \text{——} (4)$$

where ϵ_0 is the permitivity or absolute permitivity of freespace / vacuum or air and its value in SI unit is $\boxed{\epsilon_0 = 8.85 \times 10^{-12} \frac{C^2}{Nm^2}}$

~~Q. Define unit charge.~~

~~Unit charge~~

~~we have, F =~~

$\therefore$ Coulomb's law in free space / vacuum / air is given by

$$\boxed{F = 9 \times 10^9 \frac{q_1 q_2}{r^2}} \quad \text{——} (5)$$

Q. Define unit Charge.

Unit Charge

we have, $F = 9 \times 10^9 \frac{q_1 q_2}{r^2}$ (in freespace or vacuum or air)

when $q_1 = q_2 = q$, $r = 1m$, $F = 9 \times 10^9 N$, then

$$q^2 = 1 \Rightarrow q = \pm 1C$$

$\therefore$ One columb of charge (unit Charge) is defined as that Charge which when placed in freespace (vacuum) at a distance of 1meter from an equal and similar charge repels it with a force of $9 \times 10^9 N$.

Q. What is absolute permitivity / permitivity?

Permitivity: Electric permitivity is a property of a medium between two charges which determines the electric force betwen these two Charges.

Note: Electric permitivity is a measure of electric polarizability of a dielectric / electric permitivity is a measure of response of a dielectric to an external applied electric field.

A material with high permitivity polarizes more in response to an applied electric field than a material with low permitivity.

→ It is denoted by 'ε' (epsilon)

→ Its SI unit is $\frac{C^2}{Nm^2}$

→ Its Dimensional formula is

$$[\varepsilon] = [M^{-1}L^{-3}T^{4}I^{2}]$$

→ For free space or vacuum, it is denoted by ε_0 (epsilon naught).

Q. What is relative permitivity?

Relative permitivity

Relative permitivity of a medium is defined as the ratio of electric permitivity (absolute permitivity) of the medium to the electric permitivity (absolute) of the free space.

It is denoted by ε_r

$$\therefore \boxed{\varepsilon_r = \frac{\varepsilon}{\varepsilon_0}}$$

→ It is also called as dielectric constant.

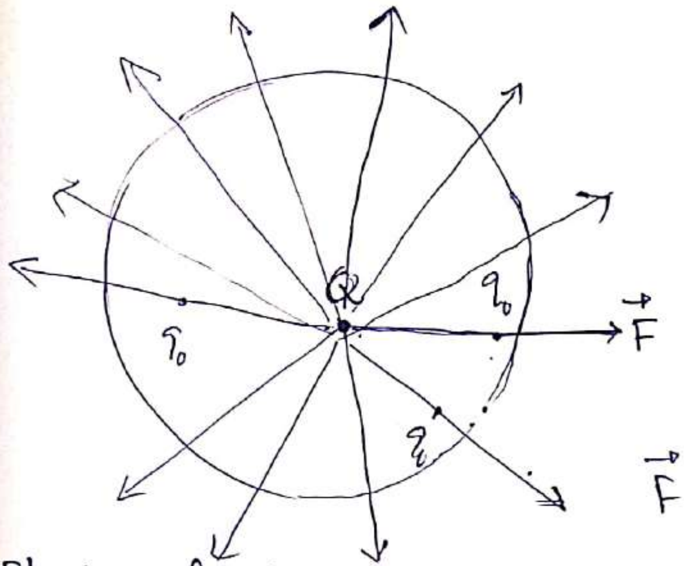
→ It has no unit and dimension.

~~Electric~~

What is electric field ?

Electric field

Electric field is a region around a charged particle within which a force (attracting or repelling) would be exerted on other charged particles.



$Q \rightarrow$ Source Charge

$q_0 \rightarrow$ test charge (positive & very small) whose electric field is very negligible

$\vec{F} \rightarrow$ electric force.

Electric field Intensity:

The electric field intensity or electric field may be defined as $\boxed{\vec{E} = \frac{\vec{F}}{q_0}}$ or $\boxed{E = \frac{F}{q_0}}$

where $\vec{F}$ is the electric force

→ Its SI unit is N/C

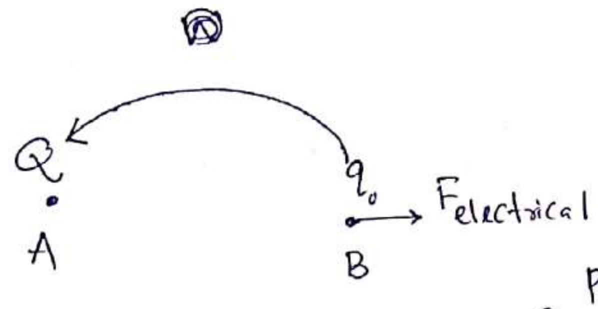
→ Its dimensional formula is $[M^1 L^1 T^{-3} I^{-1}]$

→ The electric field due to a point charge Q at a distance r is given by

$$E = \frac{\frac{1}{4\pi\epsilon_0}\frac{Q\cancel{q_0}}{r^2}}{\cancel{q_0}} \Rightarrow \boxed{E = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}} \text{ or } \boxed{E = \frac{kQ}{r^2}}$$

Q: What do you mean by electric potential difference? (7)

Electric potential difference



positive $q_0 \to$ test charge

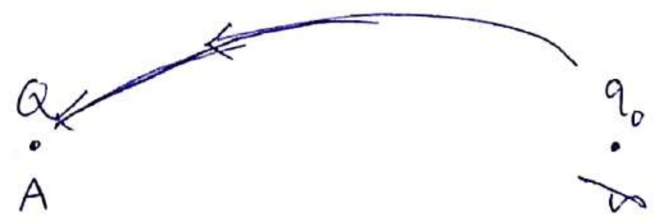
$$\boxed{V_A - V_B = \frac{W_{B\to A}}{q_0}}$$

$W_{B\to A}$ = workdone by an external force F_{ext} against the electric force $F_{electric}$ in bring the test charge q_0 from B to A very slowly (so that kinetic energy will be zero)

Q What is electric potential?

Electric potential: Electric potential at a point

Assume B is at infinite and $V_{\infty} = 0$



∴ Electric potential at point A is

$$\boxed{V_A = \frac{W_{\infty\to A}}{q_0}}$$

→ It is a scalar quantity

→ Its SI unit is J/C

→ Its dimensional formula is $[M^1L^2T^{-3}I^{-1}]$

→ Electric potential due to a point charge Q at a distance 'r' is given by

$$\boxed{V = \frac{1}{4\pi\epsilon_0}\frac{Q}{r}} \text{ (or) } \boxed{V = \frac{KQ}{r}}$$

(Q is with sign)

Q: What is a Capacitor?

Capacitor

It is an electrical device which stores electrical energy (electric charges) and supplies at sudden when needed.

Uses

Some devices like fan, motor, flash light etc require high amount of current (25A - 50A) when they start. To supply this ~~...~~ high amount of charge to these devices, we need capacitors.

Q: What is Capacitance?

Capacitance

Capacitance is the ability of a capacitor to store the electric charges or electric energy.

It is denoted by 'C' and is given by

(7)

$$\boxed{C = \frac{Q}{V}}$$

Other units of C are mF, μF, pF, nF etc. . . .

→ Its SI unit is Farad (F) $\therefore 1F = \frac{1C}{V}$

Suppose the capacitance of a capacitor is 1F its meaning is when the capacitor stores 1C of electric charge it develops a potential of 1V.

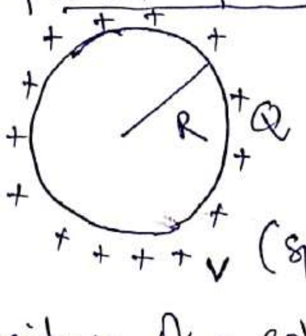
when the capacitor will give 2C of electric charge it develops a potential of 2V.

→ Capacitance (C) is independent of Q & V

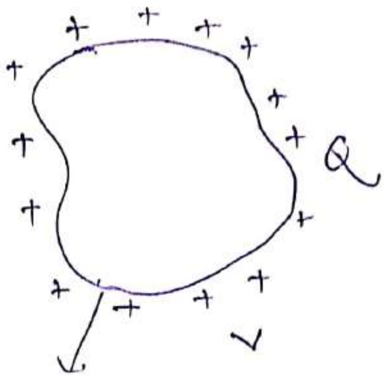
→ Capacitance depends on shape & size (dimensions) of the conductor and medium

For example :

① Spherical Capacitor



(Spherical capacitor)



A Conductor.

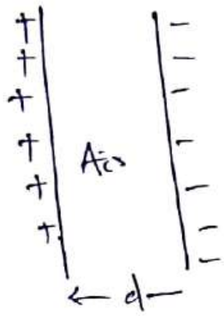
Capacitance of a spherical capacitor is

$$\boxed{C = 4\pi\epsilon_0 R}$$

R → Radius of the Sphere

ϵ_0 → permittivity of the free space (Air) as the capacitor is in free space (Air).

(2) Parallel plate capacitor



Capacitance of a parallel plate capacitor is

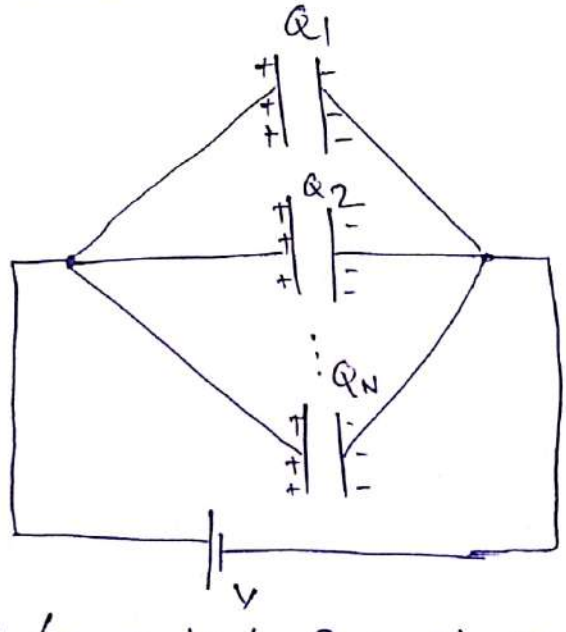
$$\boxed{C = \frac{A\epsilon_0}{d}}$$

where $A \to$ Area of the plates
$d \to$ distance between the two plates } dimensions
$\epsilon_0 \to$ permittivity of the Air } medium

Grouping of the capacitors

(i) parallel grouping (ii) Series Grouping

Parallel grouping



In parallel grouping
→ The charges on the capacitors are different
→ The potential difference across each capacitor is the same.

The total/equivalent capacitance of the grouping is given by

$$\boxed{C_p = C_1 + C_2 + C_3 + \cdots + C_N}$$

where $C_1, C_2, C_3, C_4 \cdots C_N$ are the capacitances of the capacitors

P-1

find equivalent Capacitance when two Capacitors of Capacitancy 2F & 3F are Connected in Parallel.

Solution Given $C_1 = 2F$

$C_2 = 3F$

$\therefore C_p = 2+3 = 5F$

Ans

P-2

find equivalent Capacitance when two capacitors of capacitance 2F and 3000μF are Connected in Parallel

Solution: Given $C_1 = 2F$

$C_2 = 3000\mu F = 3000 \times 10^{-6} F$

$= \frac{3000}{1000000} = \frac{3}{1000} F$

$= 0.003 F$

$\therefore C_p = 2 + 0.003 = \underline{2.003F}$ Ans

P-3

find equivalent Capacitance when three Capacitors of Capacitancy 4F, 2F and 200mF are Connected in Parallel.

Solution: Given $C_1 = 4F$

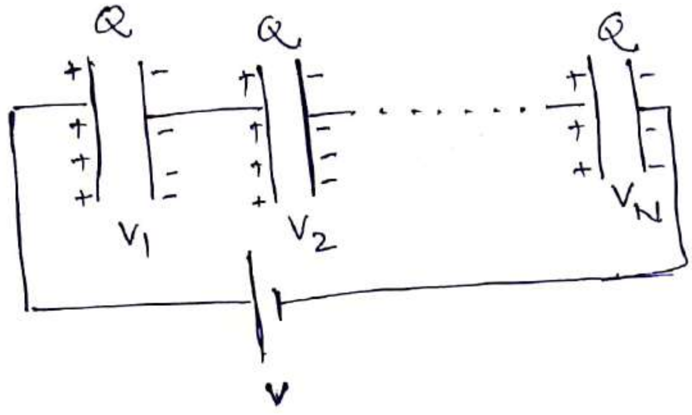
$C_2 = 2F$

$C_3 = 200mF = 200 \times 10^{-3} F = \frac{200}{1000} F$

$= \frac{2}{10} F = 0.2F$

$\therefore C_p = 4+2+0.2 = \underline{6.2F}$ Ans

Series Grouping



In Series Grouping
→ The charge on each capacitor is the same
→ The potential difference across the capacitors are different

The total/equivalent capacitance of the grouping is given by

$$\boxed{\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots + \frac{1}{C_N}}$$

where $C_1, C_2, C_3 \dots C_N$ are the capacitances of the capacitors

P-4

Find equivalent capacitance when two capacitors of capacitance 2F and 4F are connected in series.

Solution: Given $C_1 = 2F$

$C_2 = 4F$

$C_s = ?$

$$\therefore \frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$\Rightarrow \frac{1}{C_s} = \frac{1}{2} + \frac{1}{4} \Rightarrow \frac{1}{C_s} = \frac{2+1}{4} \Rightarrow \frac{1}{C_s} = \frac{3}{4}$$

$$\Rightarrow C_s = \frac{4}{3}$$

$$\Rightarrow C_s = 1.33\,F \text{ Ans}$$

P-5

find equivalent Capacitance when two capacitors of Capacitance 2f and 400mF are connected in series

Solution: Given $C_1 = 2F$

$C_2 = 400mF = 400 \times 10^{-3} F$

$= \frac{400}{1000} F = \frac{4}{10} F$.

$C_S = ?$

we have, $\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2}$

$\Rightarrow \frac{1}{C_S} = \frac{1}{2} + \frac{1}{4/10} \Rightarrow \frac{1}{C_S} = \frac{1}{2} + \frac{10}{4}$

$\Rightarrow \frac{1}{C_S} = \frac{2+10}{4}$

$\Rightarrow \frac{1}{C_S} = \frac{\cancel{12}\,3}{\cancel{4}\,1}$

$\Rightarrow \frac{1}{C_S} = \frac{3}{1}$

$\Rightarrow C_S = \frac{1}{3} \Rightarrow C_S = 0.33F$

Ans

UNIT- 09: Electrostatic & Magnetostatic

Part-02

Q: What is magnetostatic?

Magnetostatic:

Magnetostatic is the branch of physics that deals with magnets, magnetic field, magnetic forces and all the magnetic phenomena provided the currents are steady i.e not changing with time.

Q: What is a magnet?

Magnet

- Magnet is a material that has following properties

(i) Attractive property: A magnet attracts small pieces of iron, Cobalt, nickel etc.

(ii) Directive property: when a magnet is suspended or pivoted freely, it aligns itself in the geographical north - South direction.

(iii) Like poles repel & Unlike poles attract: A magnet has two poles, namely north pole (N-pole) & South pole (S-pole)

* Like poles repel

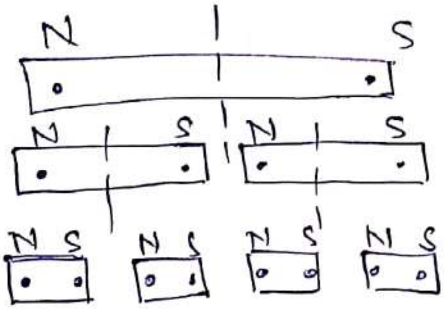
* Unlike poles attract

[S N] ← → [N S] (Repel)

[S N] ⇄ [S N] (Attract)

(iv) Magnetic monopole doesn't exist :

If we try to isolate the two poles of a magnet from each other by breaking the magnet in the middle, each broken part is found to be a magnet with N & S-poles at its ends.



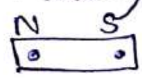
Q: ~~State and~~

Types of magnet

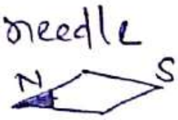
Magnet

↓ Natural magnets ⇓ Natural occuring substances that have magnetic properties e.g- Magnetite, Ferrite, etc

↓ Artifical magnets ⇓ Man made magnets with different forms

a) Bar magnet 

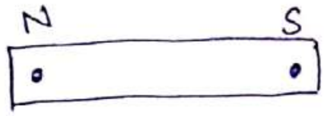
b) magnetic needle



c) Horse shoe magnet

Terms related to a magnet

(i) Poles: The ends of a magnet are called its poles. One end is called north pole, the other is called south pole. A pole is characterized by pole strength, denoted by 'm'.

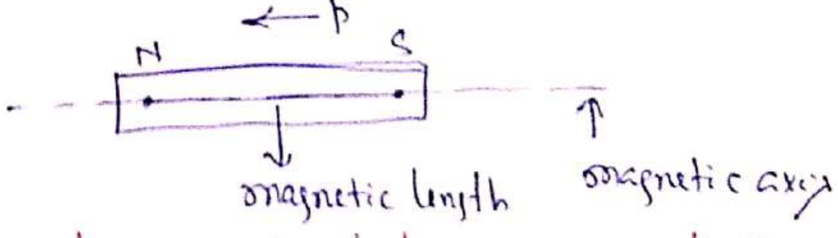
(ii) 

(ii) Magnetic axis: The line passing through the two poles of a magnet is called magnetic axis

(iii) magnetic length: The distance between the two poles of a magnet is called magnetic length. It is denoted by 'l' or '$2l$'

(iv) magnetic moment: It is the product of pole strength & magnetic length of a magnet. It is a vector quantity whose direction is from south pole to north pole of a bar magnet. It is denoted by $\vec{p}$

$$\therefore \vec{p} = m\vec{l} \quad \text{(or)} \quad p = ml$$



Q: State & explain Coulomb's law in magnetism.

Coulomb's law in magnetism:

Statement: This law states that the magnetic force between two poles is

→ directly proportional to the product of their pole strengths

→ inversely proportional to the square of distance between two poles

Explanation

Let F be the magnetic force between two poles of pole strengths m_1 & m_2, ~~placed at~~ and r be the distance between them

According to Coulomb's law

$$F \propto m_1 m_2 \quad \text{---(1)}$$

$$F \propto \frac{1}{r^2} \quad \text{---(2)}$$

Combining eqn (1) & eqn (2), we get

$$F \propto \frac{m_1 m_2}{r^2}$$

or, $$\boxed{F = K \frac{m_1 m_2}{r^2}} \quad \text{---(3)}$$

where K is a proportionality constant and its value depends on following two factors

(i) the nature of the medium between

(ii) the system of units chosen to measure F, m_1, m_2 & r

when the medium is air or free space, ~~[illegible]~~ ~~[illegible]~~ then

$$\boxed{F = \frac{\mu_0}{4\pi}\frac{m_1 m_2}{r^2}} \quad \text{——— (4)}$$

where μ_0 is the permiability of freespace or air and its ~~[illegible]~~

ue in is unit ~~[illegible]~~ $\mu_0 = 4\pi \times 10^{-7}\ Wb\, A^{-1} m^{-1}$

Q: Define unit pole.

Unit Pole

we have $F = \frac{\mu_0}{4\pi}\frac{m_1 m_2}{r^2}$

when $m_1 = m_2 = m$, $F = 10^{-7}$ N and $r = 1m$, then

$$10^{-7} = \frac{4\pi \times 10^{-7}}{4\pi}\frac{m^2}{1^2}$$

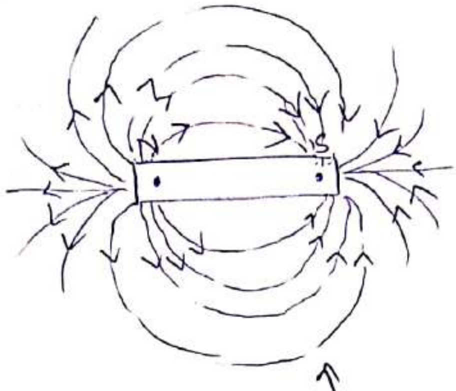
$$\Rightarrow \boxed{m = \pm 1}$$

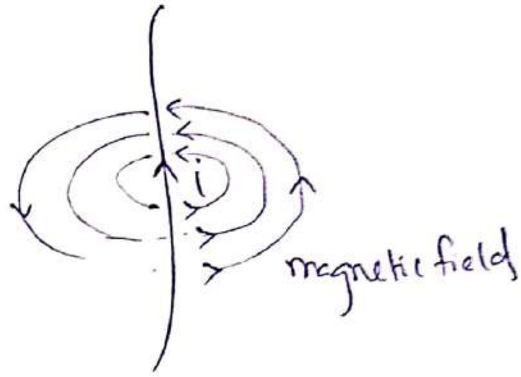
Definition: A unit pole is that pole which when placed, in vacuum, at a distance of 1m from a similar pole repels it with a force of 10^{-7} N

Q: Define magnetic field.

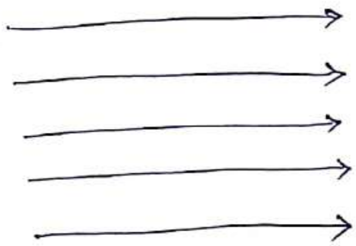
Magnetic field

Magnetic field is a region around a magnet or a current carrying conductor within which a force would be exerted on other magnets or moving charged particles.

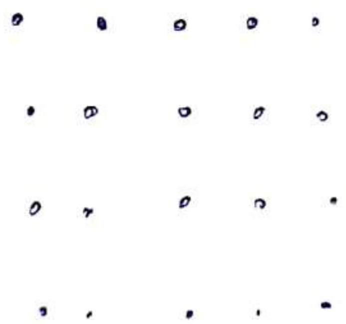


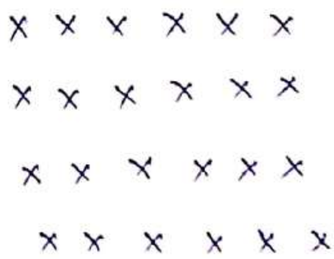


Representation of a magnetic field



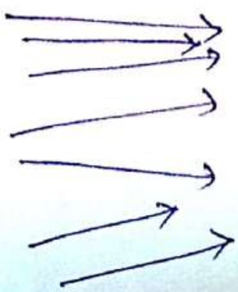
Uniform magnetic field directed from left to right





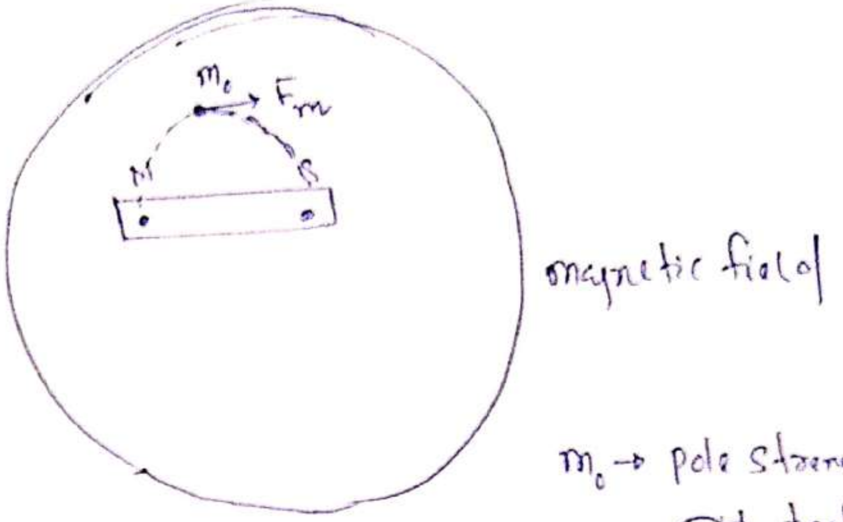
uniform magnetic field directed outwardy

Uniform magnetic field directed inwardly



Non uniform magnetic field.

Definition of magnetic field intensity or magnetic field ($\vec{B}$)



$m_0 \to$ pole strength of a test north pole which is very small.

(i) The magnetic field intensity may be defined as

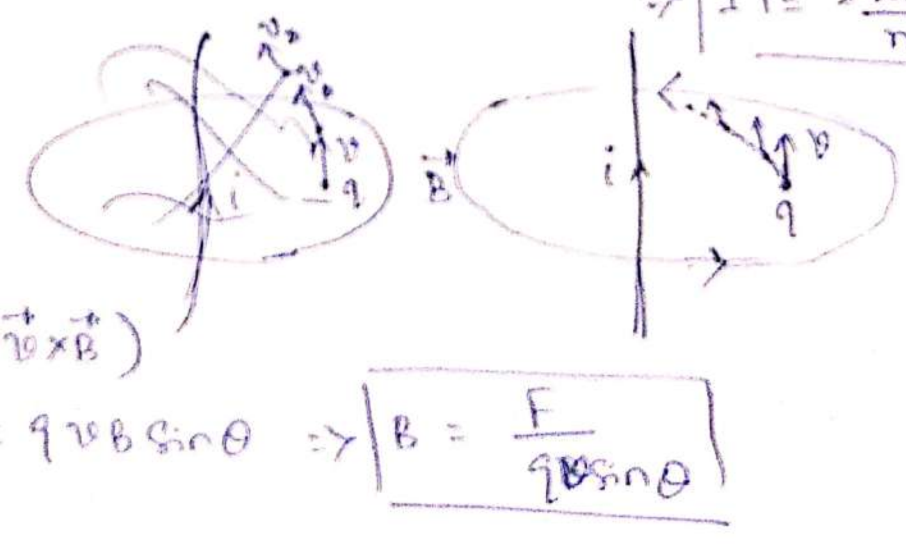
$$\vec{B} = \frac{\vec{F}_m}{m_0}$$

or, $\boxed{B = \frac{F_m}{m_0}} \Rightarrow \boxed{B = \frac{\mu_0}{4\pi}\frac{m}{r^2}}$

$\therefore$ SI unit is $\underline{NA^{-1}m^{-1}}$ or $\boxed{\text{Tesla (T) or } \frac{Wb}{m^2}}$

$\Rightarrow \boxed{1\,T = 1\,\frac{Wb}{m^2}}$

(ii)



$\vec{F} = q(\vec{v} \times \vec{B})$

(or) $F = qvB\sin\theta \Rightarrow \boxed{B = \frac{F}{qv\sin\theta}}$

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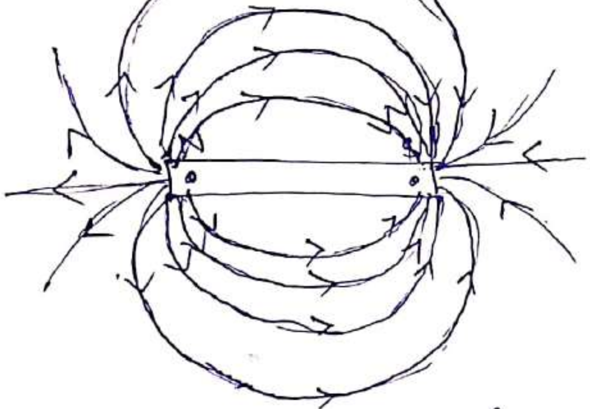
Q: Define magnetic lines of force

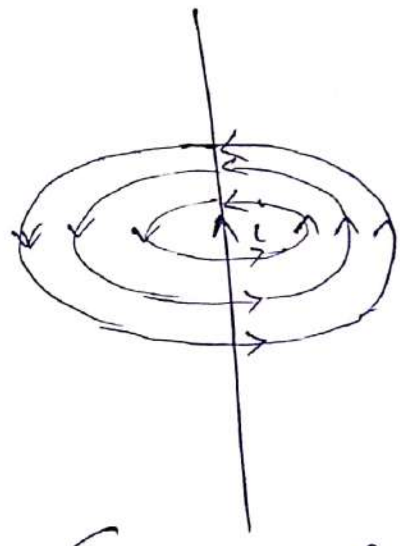
Magnetic lines of force

Magnetic lines of force are the paths taken by a test north pole if it were ~~free to~~ completely free to move under the action of the magnetic force inside a magnetic field

→ These are imaginary lines drawn to represent ~~the~~ a magnetic field



{magnetic lines of force for a bar magnet}



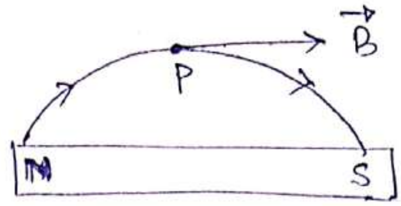
{magnetic lines of force for a current carrying conductor}

Q: write properties of magnetic lines of force

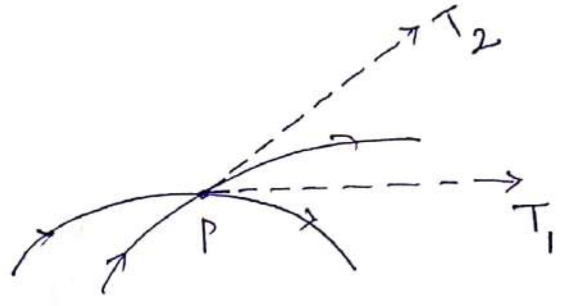
Properties of magnetic lines of force

→ Magnetic lines of force always starts from a north pole and ends at a south pole

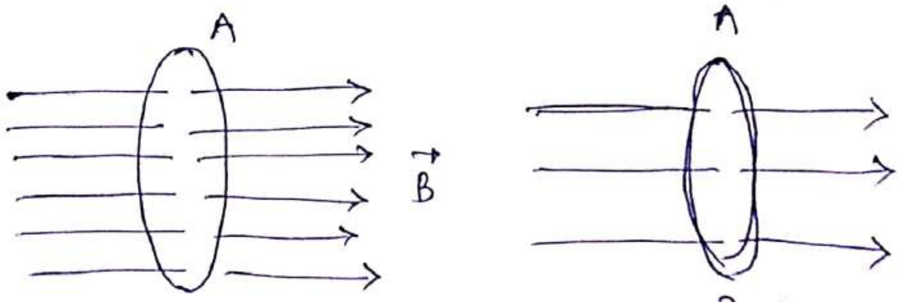
→ Tangent, at any point to the magnetic line of force gives direction of the magnetic field at that point.



→ Two lines of force never cross each other.



→ The no. of lines of force per unit area (area being perpendicular to the lines) is proportional to the strength of the magnetic field



No. of lines of force per unit Area is more, so magnetic field is strong

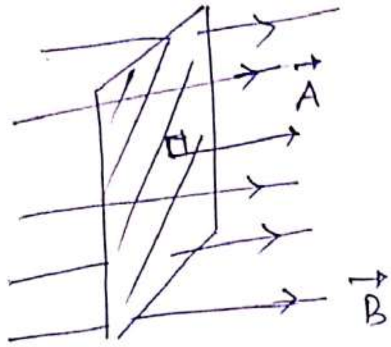
No. of lines of force per unit area is less, so magnetic field is weak

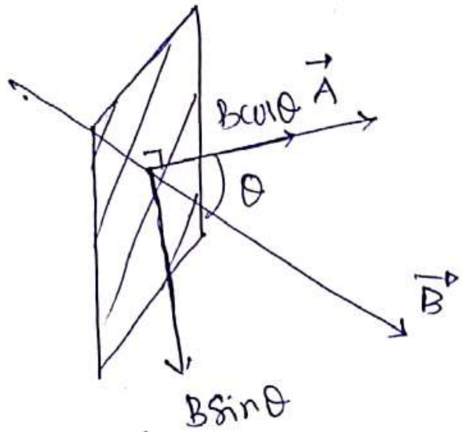
Q: Define magnetic flux

Magnetic flux

magnetic flux is the no. of ~~loso~~ magnetic lines of force passing perpendicular to the area of the surface or plane of the surface.

~~The magnatic~~





Magnetic flux linked with a surface is given by

$$\phi_B = \vec{B}\cdot\vec{A}$$

or, $\boxed{\phi_B = BA\cos\theta}$

A is the area of the surface.

θ is the angle between $\vec{B}$ & $\vec{A}$

→ It is a scalar quantity

→ Its SI unit is Weber (Wb)

→ when $\theta = 90°$, $\phi_B = BA \Rightarrow \boxed{B = \frac{\phi_B}{A}}$ and is called flux density i.e magnetic lines per unit area, provided area is perpendicular to the flux lines.

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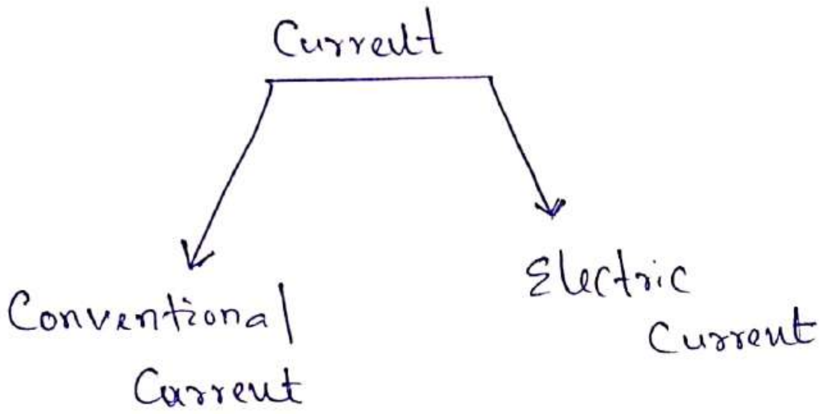
Unit-10: Current Electricity

Q: What is Current?

Current

Current is the flow of charges in a circuit. and is defined as

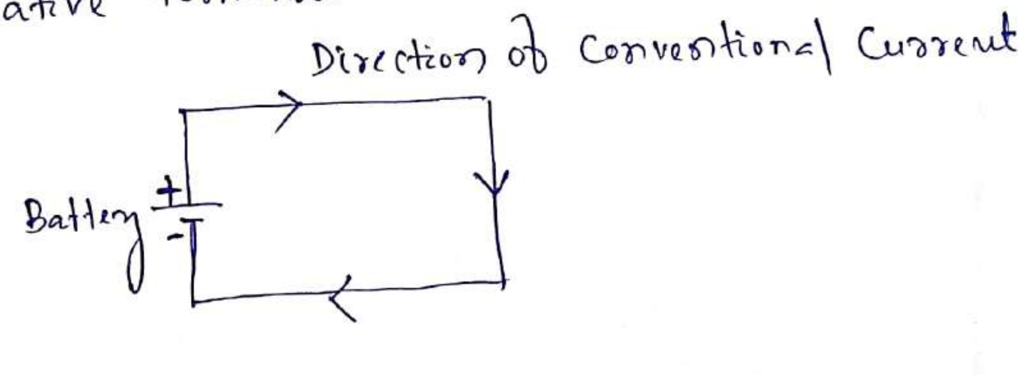
$$\boxed{\text{Current} = \frac{\text{Charge}}{\text{time}}}$$



Conventional Current

⇒ Conventional Current is the flow of positive charges in a circuit.

⇒ The direction of conventional current is from positive terminal to the negative terminal

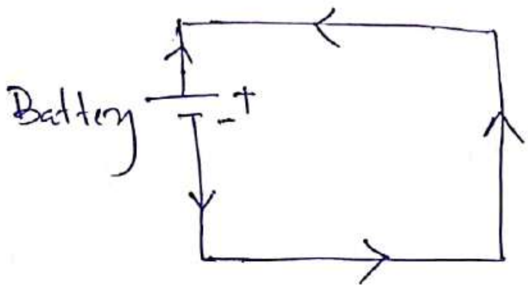


Q: What is electric Current?

Electric Current

Electric Current is the flow of negative charges (electrons) in a circuit

⇒ The direction of electric current in a circuit is from negative terminal to the positive terminal



⇒ It is denoted by 'i' or 'I' and therefore, $\boxed{i = \frac{q}{t}}$

⇒ It is a fundamental physical quantity

q → electric charge
t → time

⇒ Its SI unit is Ampere (A) and

$$\boxed{1 \text{ Ampere} = \frac{1 \text{ Coulomb}}{1 \text{ Second}}}$$

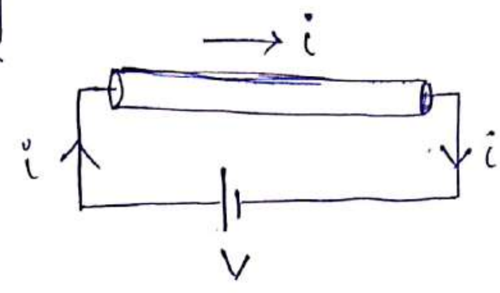
Q: What is Ohm's law?

Ohm's law

This law states a relation between electric current (i), electric potential difference (V) and resistance (R).

Statement: This law states that electric current in a conductor is directly proportional to the electric potential difference between the ends of the Conductor. ③

Mathematically, $\boxed{i \propto V}$



$\Rightarrow \boxed{i = \frac{V}{R}}$

$\Rightarrow \boxed{V = iR}$

Where R is a proportionality Constant and is called resistance of the Conductor.

Q: Write applications of ohm's law.

Applications of Ohm's law

→ To determine the voltage, resistance or current of an electric Circuit

→ To maintain the desired voltage drop across the electronic Components

Q: What is a resistor?

Resistor

→ Resistor is an electrical component that opposes/reduces the electric Current passing through it.

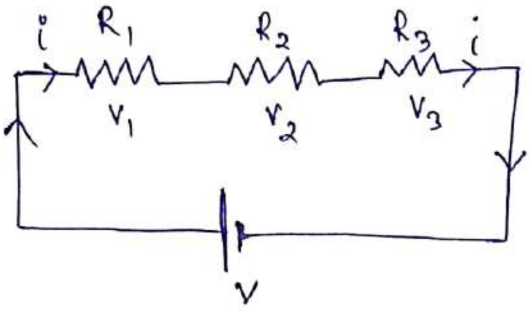
⇒ The ability of ~~the~~ a resistor to reduce the current is Called resistance (R)

⇒ ~~Resist~~ The SI unit of resistance (R) of a resistor is Ohm (Ω).

⇒ Other units of resistance are $m\Omega$ (mili ohm) and $\mu\Omega$ (micro ohm)

⇒ Circuit Symbol of a resistor is 

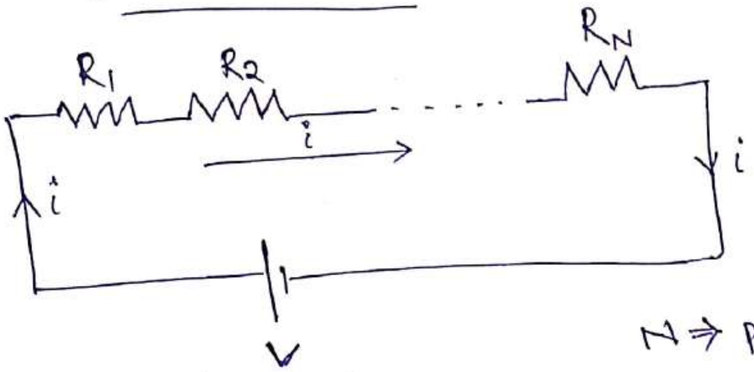
Series Combination of resistors



In Series Combination
* The current through each resistor is the same.
* The potential difference across each resistor is different.

Formula for equivalent resistance is

$$\boxed{R_s = R_1 + R_2 + R_3}$$



N ⇒ positive integer

Formula for equivalent resistance is

$$\boxed{R_s = R_1 + R_2 + R_3 + \dots + R_N}$$

Problem-01

Two resistors of resistance 2Ω and 3Ω are Connected in Series. Find equivalent resistance.

Soln: Given $R_1 = 2\Omega$

$R_2 = 3\Omega$

$\therefore R_s = R_1 + R_2 = 2 + 3 = 5\Omega$

Problem-02

five resistor of resistance 2Ω each are connected in series, find equivalent resistance.

Soln: Given $R_1 = R_2 = R_3 = R_4 = R_5 = 2\,\Omega$, $R_s = ?$

$$\therefore R_s = R_1 + R_2 + R_3 + R_4 + R_5 = 2+2+2+2+2 = 10\,\Omega$$

Problem-03

Two resistors of resistance 2Ω and 4 mΩ are connected in series. find equivalent resistance.

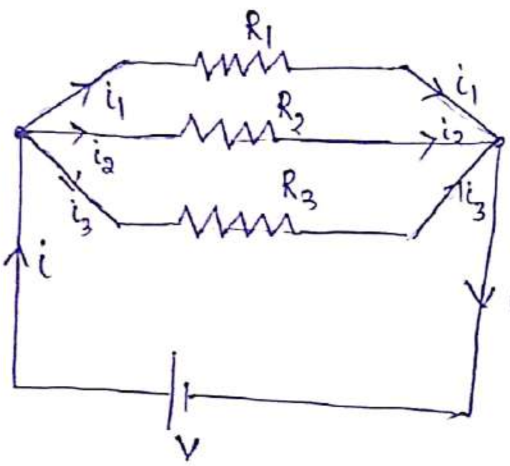
Soln: Given $R_1 = 2\,\Omega$

$R_2 = 4\,m\Omega = 4 \times 10^{-3}\,\Omega = 4 \times \frac{1}{1000}\,\Omega$

$R_s = ?$ $\quad = \frac{4}{1000}\,\Omega = 0.004\,\Omega$

$$\therefore R_s = 2 + 0.004 = \underline{2.004}\,\Omega$$

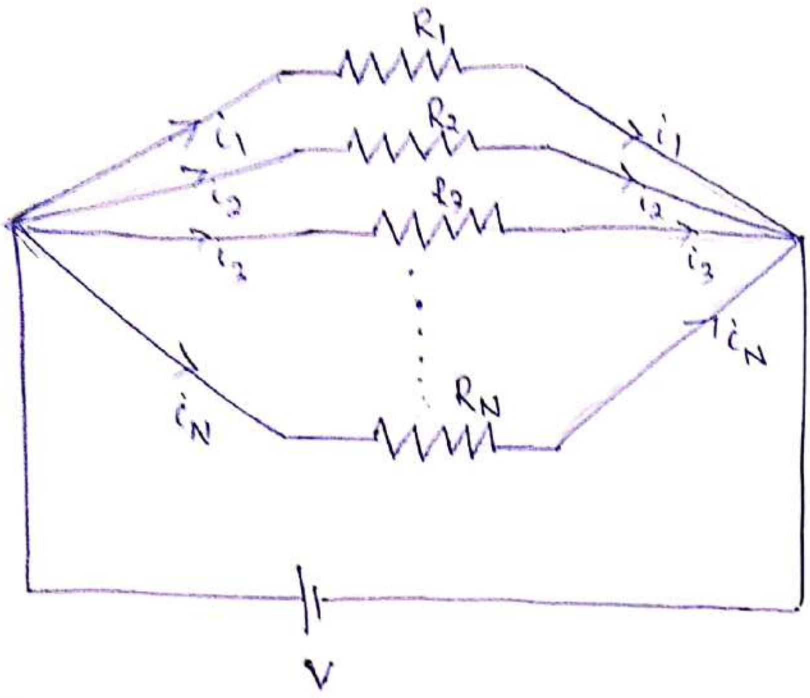
Parallel Combination of resistors



In parallel combination
* The current flowing through each resistor is different
* The potential difference across each resistor is the same

Formula for equivalent resistance is

$$\boxed{\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$



The formula for equivalent resistance is

$$\boxed{\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots + \frac{1}{R_N}}$$

Problem-04

Find the equivalent ~~restance~~ resistance when two resistors of resistance 2Ω and 3Ω are connected in parallel.

Soln: Given $R_1 = 2\Omega$

$R_2 = 3\Omega$

$\therefore R_p = ?$

we have, $\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} \Rightarrow \frac{1}{R_p} = \frac{1}{2} + \frac{1}{3} \Rightarrow \frac{1}{R_p} = \frac{3+2}{6}$

$\Rightarrow \frac{1}{R_p} = \frac{5}{6}$

$\Rightarrow R_p = \frac{6}{5} = 1.2\Omega$

$\therefore \boxed{R_p = 1.2\Omega}$

Vimp

Q: State and Explain Kirchhoff's laws

Kirchhoff's law

There are two laws or rules proposed by German Physicist as follows

1st law / Junction law / Current law (KCL)

Statement: The algebric sum of currents meeting at a point or a Junction in a circuit is equal to zero

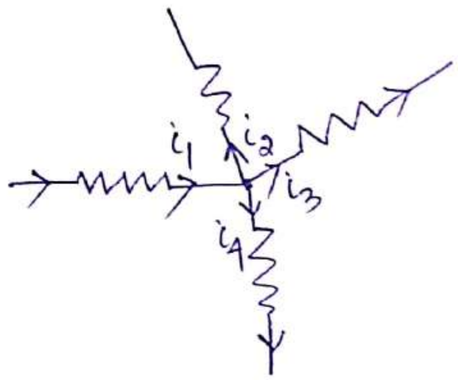
i.e $\boxed{\sum i = 0}$

Sign of Convention

⇒ The current entering to the junction is Considered as positive

⇒ The Current leaving the junction is Considered as negative

Explanation



Here, $i_1 + (-i_2) + (-i_3) + (-i_4) = 0$

$\Rightarrow i_1 - i_2 - i_3 - i_4 = 0$

$\Rightarrow \boxed{i_1 = i_2 + i_3 + i_4}$

⇒ Sum of current entering the junction = Sum of the Currents leaving the Junction.

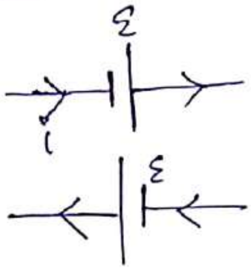
2nd law / Loop law / voltage law (KVL)

Statement: The algebric sum of all the potential differences along a closed loop in a circuit is equal to zero.

i.e $\boxed{\Sigma V = 0}$

Sign of Convention

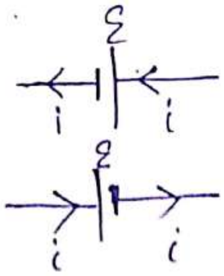
⇒ If the electric current flows through a battery from negative terminal to the positive terminal, then emf is taken as positive



$\boxed{\varepsilon = +ve}$

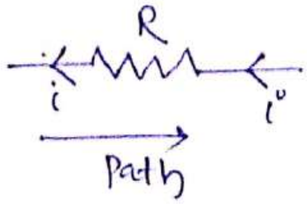
$\varepsilon = +ve$

⇒ If the electric current flows through a battery from positive terminal to the negative terminal, then emf is taken as negative



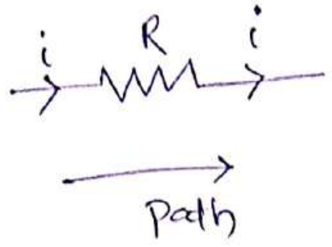
$\boxed{\varepsilon = -ve}$

⇒ If the path taken is opposite to the direction of current, then potential difference across a resistor is ~~take~~ consider as positive



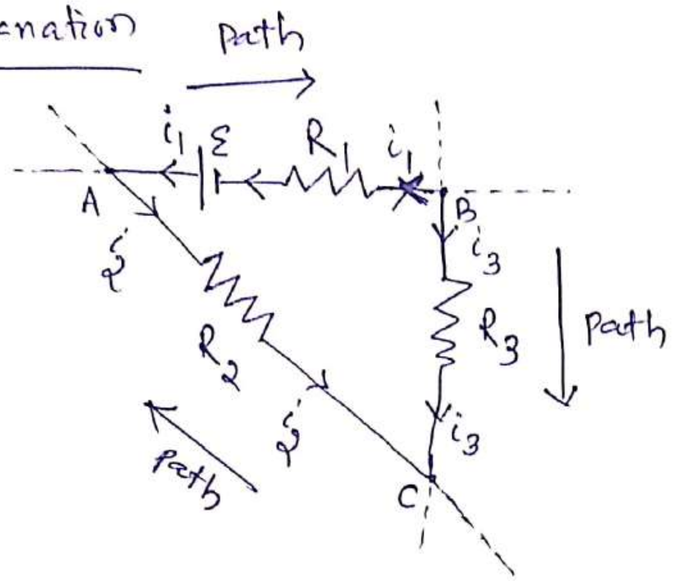
$\boxed{iR = +ve}$

⇒ If the path taken and the direction of the current is the same, then potential difference across a resistor is considered as negative



$\boxed{iR = -ve}$

<u>Explanation</u>



Applying KVL to the closed loop ABCA

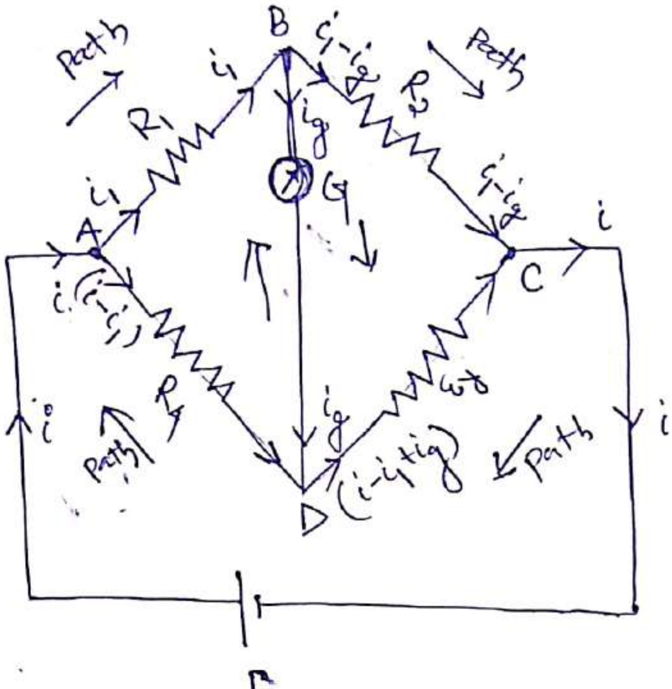
$$\varepsilon + i_1R_1 + (-i_3R_3) + i_2R_2 = 0$$

$$\Rightarrow \varepsilon + i_1R_1 - i_3R_3 + i_2R_2 = 0$$

Q: Derive balance condition for a wheatstone Bridge by applying Kirchhoff's laws

<u>Balance Condition for a Wheatstone Bridge</u>

A wheatstone bridge is an electrical arrangement which used to determine unknown resistance.



* There are four resistors of resistances R_1, R_2, R_3 & R_4 and are connected to form a loop.

* A battery is connected between the junctions A & C

* A Galvanometer of resistance 'G' is connected in between the junctions B & D

Applying KVL to the closed loop ABDA,

$$-i_1R_1 + (-i_gG) + (i-i_1)R_4 = 0$$

$$\Rightarrow -i_1R_1 - i_gG + (i-i_1)R_4 = 0 \quad \text{--- (1)}$$

Similarly, Applying KVL to the closed loop BCDB,

$$-(i_1-i_g)R_2 + (i-i_1+i_g)R_3 + i_gG = 0 \quad \text{--- (2)}$$

It is noted that, a wheatstone bridge is said to be balanced when $\boxed{i_g=0}$

Substitute, $i_g=0$ in eqn (1) and eqn (2)

$$-i_1R_1+(i-i_1)R_4=0 \quad \left[\text{from eqn (1)}\right]$$

$$\Rightarrow \cancel{-}i_1R_1=\cancel{-}(i-i_1)R_4$$

$$\Rightarrow i_1R_1=(i-i_1)R_4 \quad \text{—— (3)}$$

Again, $-(i_1-0)R_2+(i-i_1+0)R_3 \pm 0=0$

$$\Rightarrow -i_1R_2+(i-i_1)R_3=0$$

$$\Rightarrow \cancel{-}i_1R_2=\cancel{-}(i-i_1)R_3$$

$$\Rightarrow i_1R_2=(i-i_1)R_3 \quad \text{—— (4)}$$

Dividing eqn (3) by eqn (4), we get

$$\frac{\cancel{i_1}R_1}{\cancel{i_1}R_2}=\frac{\cancel{(i-i_1)}R_4}{\cancel{(i-i_1)}R_3}$$

$$\Rightarrow \boxed{\frac{R_1}{R_2}=\frac{R_4}{R_3}} \quad \text{—— (5)}$$

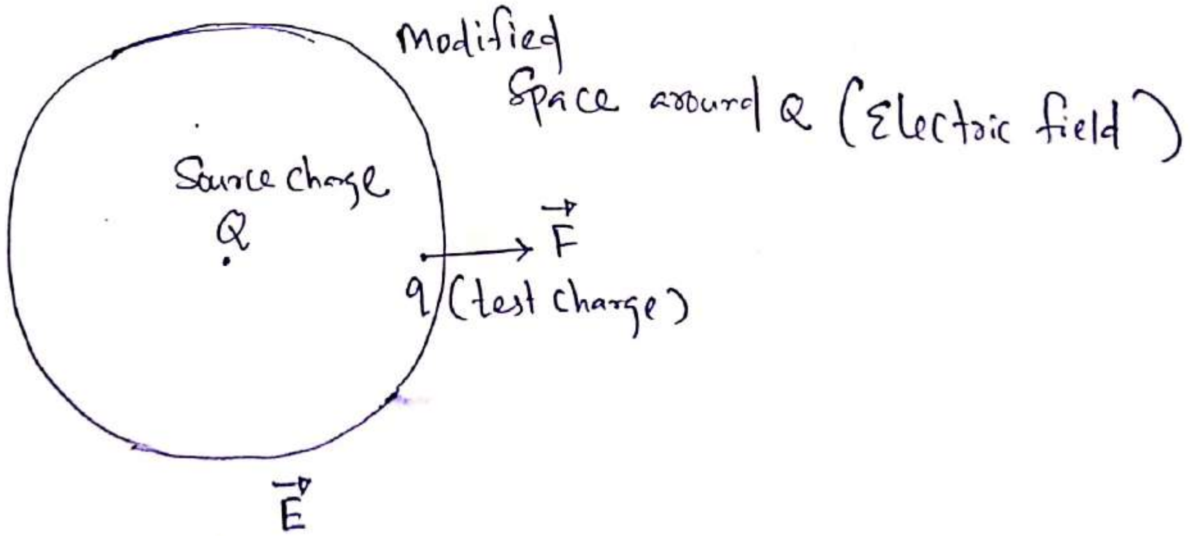
This is the balance condition for a Wheatstone Bridge.

Unit-II: Electromagnetism & Electromagnetic Induction

Initially, It was believed that ~~there was~~ electricity & magnetism are two different topics and there was no interlink between them.

Later from work of Faraday, maxwell, it was confirmed that there is a interlink between electricity and magnetism as a changing magnetic field induces current in a wire loop where as a changing electric field ~~induce~~ produce magnetic field.

Electric field

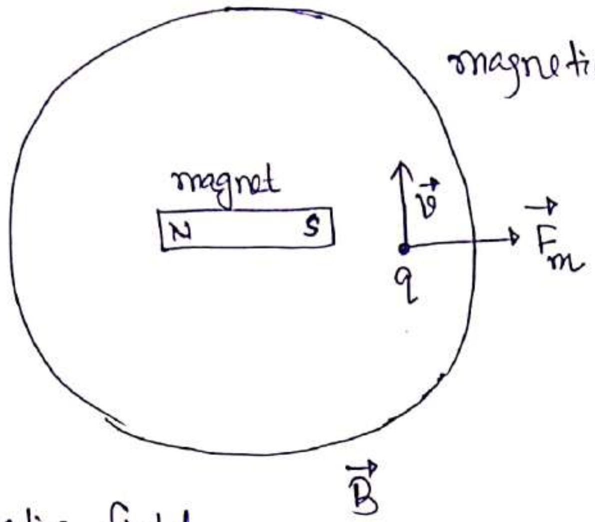


$\vec{E} \rightarrow$ electric field

$\vec{F_e} \rightarrow$ electric force

$\vec{E} = \dfrac{\vec{F_e}}{q}$ or $\boxed{\vec{F_e} = q\vec{E}}$

Magnetic field



$\vec{B} \rightarrow$ Magnetic field

$\vec{F}_m \rightarrow$ magnetic force

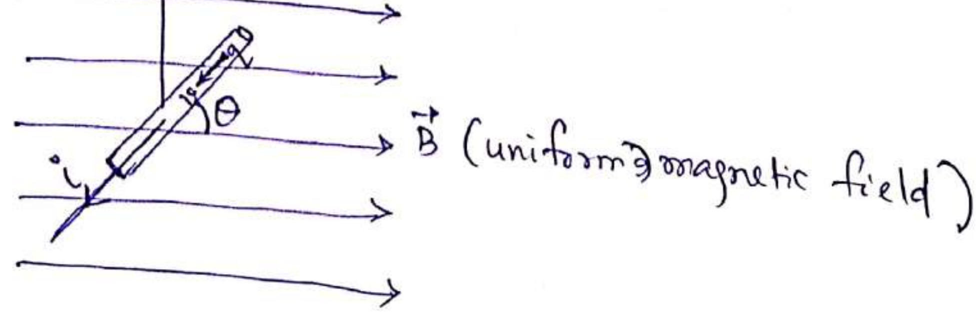
$$\boxed{\vec{F}_m = q(\vec{v} \times \vec{B})}$$

$\Rightarrow \vec{F}_m = qvB\sin\theta\,\hat{n}$

$\therefore$ magnitude of $\vec{F}_m$ is $F_m = qvB\sin\theta$

Q: Derive an expression for the force acting on a current carry conductor placed in a uniform magnetic field.

Force acting on a current carrying conductor placed in a uniform magnetic field



Consider a Conductor of length 'L' is placed in a uniform magnetic field $\vec{B}$

we have, $\vec{F}_m = q(\vec{v} \times \vec{B})$

or, $\vec{F}_m = q\,vB\sin\theta\,\hat{n}$

or, $\vec{F}_m = q\,\frac{L}{t}B\sin\theta\,\hat{n}$

or, $\vec{F}_m = \frac{q}{t}LB\sin\theta\,\hat{n}$

or, $\vec{F}_m = i\,LB\sin\theta\,\hat{n}$

where $\frac{q}{t} = i =$ Current flowing in the conductor

or, $\boxed{\vec{F}_m = i(\vec{L} \times \vec{B})}$

Magnitude of $\vec{F}_m$: $|\vec{F}_m| = F_m = iLB\sin\theta$

Direction of $\vec{F}_m$: Direction of $\vec{F}_m$ Can be determined by using Flemming's left hand Rule (FLHR)

Q: State Faraday's laws of Electromagnetic Induction.

Faraday's laws of electromagnetic Induction

Faraday's law deal with the induced emf (Current) in an electrical circuit when magnetic flux linked with the circuit changes.

There are three laws proposed by Faraday as follows (4)

First law

Whenever magnetic flux linked with a circuit changes, an emf is induced in it.

Second law

The induced emf exists in the circuit so long as the change in magnetic flux linked with the circuit continues.

Third law

The induced emf is directly proportional to the negative rate of change of magnetic flux linked with the circuit.

Therefore, $\boxed{\mathcal{E} \propto -\frac{d\phi_B}{dt}}$

where $\mathcal{E} \rightarrow$ Induced emf

$\phi_B \rightarrow$ magnetic flux

$\frac{d\phi_B}{dt} \rightarrow$ Rate of change of magnetic flux

or, $\boxed{\mathcal{E} = -K\frac{d\phi_B}{dt}}$

where K is a proportionality constant.

NB: Negative sign is due to the direction of induced emf (current)

Q: What is Lenz's law?

Lenz's law

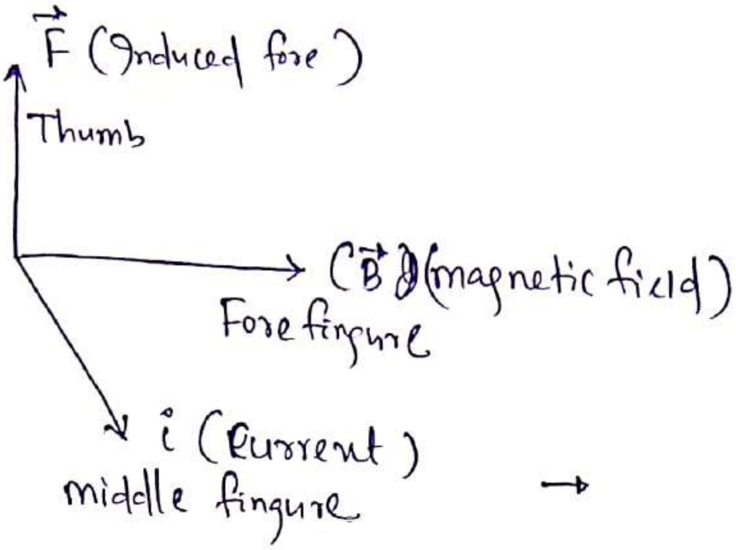
This law deals with the direction of induced emf or current in an electric circuit due to a change in magnetic flux linked with the electric circuit.

Statement: It states that the direction of induced emf is such that it tends to oppose the cause which produces it.

Q: Distinguish between Fleming's left hand rule (FLHR) and Fleming's Right Hand Rule (FRHR)

Flemings Left Hand Rule	Fleming's Right Hand Rule
→ It gives direction induced force on a current carrying conductor placed in a uniform magnetic field	→ It gives direction of induced emf in a electric circuit due to change in magnetic flux linked with the circuit.
→ This law states that if the fore finger gives direction of magnetic field ($\vec{B}$) and middle finger gives direction of current (i) in a conductor, then the induced force ($\vec{F}$) is given by thumb of left hand	→ This law states that if the fore finger gives direction of $\vec{B}$ and the thumb gives direction of applied force ($\vec{F}$) then the direction of induced emf is given by middle finger

→



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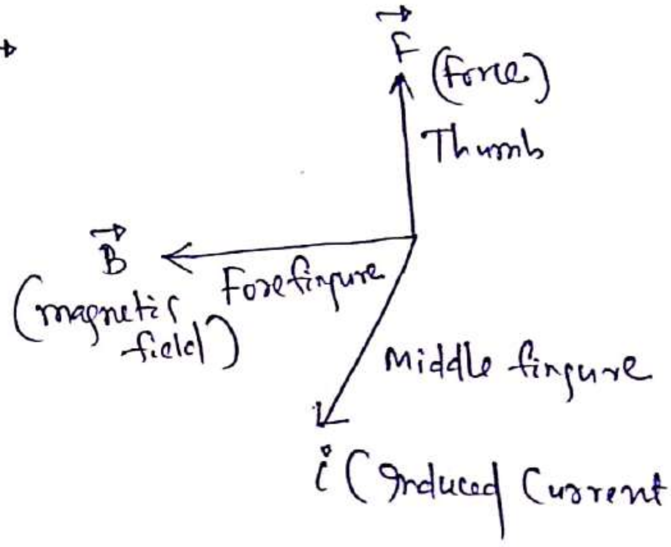

→ It is applied to motors

→ It is applied to Generators